8.

A
MISCELLANY
OF
MATHEMATICKS
IN
Two PARTS.

By RICHARD LOCKE.

Μεγα Βιβλιον, μεγα κακον.

——— Nec quicquam prodeſt
Aerias tentaſſe domos, animoque rotundum
Percurriſſe Polum morituro.
Hor.

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John Earl of Bute. &c

TO THE READER.

ACCORDING to uſual Cuſtom, it is neceſſary for me to ſay ſomething by Way of Introduction, in order to ſhew the Reader what he is to expect in this Book. The Squaring the Circle was publiſhed ſometime ſince, as well as ſome other Parts, only there was an Error in the Circle, by making two Surds inſtead of an infinite one, beſides the Surds not rightly ordered, the Fault is owned, and now I challenge all the World to confute it; and if it is true that the *Dutch* have offered ten thouſand Pounds for the Squaring the Circle, I hope they will either pay the Money, or get ſome one to demonſtrate that it is not done. The Reflections naturally flow from the Circle, and, as far as I know, moſt of them new. What is ſaid about the Equations, I am apt to think, will be found more regular than what has yet been given, becauſe they have not made a Diſtinction between the Equation and Velocity: I ſhall leave the Reader to judge of the other Parts as he thinks fit, but I hope he will give them a ſecond Reading, before he condemns them.

As for the ſecond Part 'tis moſtly new, tho' nothing is advanced but what is founded on the Principles of *Euclid*; the *Longitude* muſt be left to Time and Experience; and if the Method contain'd in this Book will not do, I am ſure there is no other. As for finding the Moon's Place, I believe the Method propoſed here is as good as any; it has been tried with Succeſs, and I could wiſh it may be try'd by a skilful Obſerver, according to the Method propoſed of the Lunar Line, Equation and Velocity; and I doubt not but it will ſave many tedious Operations and Obſervations.

Ne parva averſeris; ineſt ſua gratia parvis.

I neither write for Ambition or Intereſt, but I have other Things to purſue, that are of an infinitely greater Conſequence, which is the Salvation of the Soul; the Goods of Life are but for a ſhort uncertain Moment, but of the Soul for Eternity: I could have wrote it in *Latin*, but I preferr'd my own Country Language, for every one now is capable of examining it. Reader, farewel, and pray for me, which is the greateſt Recompence I can expect for my ſeven Years Study and Obſervations in compoſing and finiſhing this little Book.

R. LOCKE.

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A

A
MISCELLANY
OF
MATHEMATICKS.

PART I.

CHAP. I.

Aſtronomical DEFINITIONS.

1. IN order to make ſome Things more clear, as well as more uſeful, it will be neceſſary, by way of Introduction, to give a ſhort general Notion of Aſtronomy, or of the Nature of the Sphere and its Circles.

2. A Sphere, according to the Definition of *Euclid*, is form'd by the Revolution of a Semicircle round the Diameter, as *P q p* round *P p* (*Fig.* 5. *Plate* 2.)

3. The Center of the Sphere is the Center of the Diameter, as *C.*

4. The Poles are the two Points which terminate at the two Ends of the Axis, or Diameter, as *Pp.*

5. Aſtronomers, for the better underſtanding the Motions of the heavenly Bodies, have plac'd imaginary Circles upon the Earth, which are the ſame in the Heavens, and ſome are fix'd, and ſome moveable, beſides greater and leſſer ones.

6. A greater Circle divides the Sphere into two equal Parts, and has the ſame Center with the Sphere, and conſequently all great Circles cut one another into two equal Semicircles.

7. The Equator or Equinoctial Circle, the Ecliptick and two Solſtices, are great Circles which are fix'd or unchangeable.

8. The Equator *e q*, divides the Earth and World into two equal Parts, North and South; and the two Poles of the World are juſt 90 Degrees diſtant.

9. The Ecliptick *t t*, is deſcrib'd by the Earth's, or Sun's annual Motion, and 'tis probable this was the Circle it always moved in; for if the Sun ever mov'd only upon the Equator, it could not diſtinguiſh the Seaſons of the Year as mentioned, *Gen.* i. 14. And 'tis call'd the *Ecliptick*, becauſe in or near it are the Eclipſes of the Sun or Moon; its greateſt Declination from the Equinoctial is 23° 30′, or rather Northward 23° 30′ 50″, and Southward 23° 29′ 10″, the Reaſon of which will be given afterwards. It is likewiſe called the *Zodiack*, that is, ſuppoſing it to be taken about eight Degrees broad on each Side of the Ecliptick Line, becauſe in that Compaſs all the Planets are ſeen to perform their Revolution; and Aſtronomers have divided it into twelve Parts, calling them moſtly by ſome living Animal, as *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, *Piſces*, each containing thirty Degrees, which multiplied into the twelve Signs, is 360 Degrees.

10. The two Solſtices *P p* and *P q p e*, divide the whole Sphere from Eaſt to Weſt, into four equal Parts, which

meaſure out the four Seaſons of the Year.

11. The ſmaller Circles that are fix'd, are the two Tropicks, and the Polar Circles.

12. The Tropicks of *Capricorn* and *Cancer*, *e t* and *t c*, are the two Circles which terminate the greateſt Diſtance of the Ecliptick from the Equator, = 23° 30′.

13. The two Polar Circles, *Artick* and *Antartick*, *n a* and *ſ a*, are at an equal Diſtance from the Poles of the World = 23° 30′, and are the Circles of the Poles of the Ecliptick.

14. The changeable Circles are the Horizon, Meridians, and Circles of Latitude; every the leaſt Alteration of Poſition alters the Horizon, which terminates the Land or Sea, and Sky; and conſequently two Perſons cannot have the ſame Horizon; ſo likewiſe every different Situation alters the Longitude or Latitude; if to the North or South, it is the Latitude; if to the Eaſt or Weſt, it is the Longitude; and conſequently two Perſons cannot have the ſame Noon, unleſs they ſtand directly North and South, neither the ſame Latitude, unleſs they ſtand Eaſt and Weſt.

15. Every Spectator, in whatever Part of the World he is, is in the Center of his View, for when he looks up at the Heavens, he always ſees them as a Concave or Spherical Surface; and the ſame Proſpect of the Heavens will appear whether the Earth goes round the Sun, or the Sun round the Earth, becauſe the Diſtance between the Earth and the Sun is ſo little, in Compariſon of the Diſtance of the fixed Stars, that it is but a Point, which is the Reaſon that the fixed Stars, as to Bigneſs, are always the ſame, and are very little or nothing elevated or depreſſed in the annual Parallax, as they would be if there was any viſible Diſtance.

16. That the Earth moves round the Sun in its annual Revolution, as well as round its Axis in the diurnal, is the moſt probable Opinion.

17. For if the Earth doth not turn round its Axis in twenty four Hours, 'tis certain the whole Heavens muſt be whirled round in that Time; but the Velocity will be ſo great, that it renders it not only improbable, but impoſſible for the Progreſſive Motion of Matter: For, to inſtance in the Dog-ſtar, which according to Mr. *Roberts*, is 480000000000 Miles diſtant from the Earth, conſequently,

quently, the Circumference of its Revolution in round Numbers, muſt be above ſix times as much, or 288000000000 Miles; divide this by 5184000, the Thirds of a Day, and it gives 555555 Miles in a Third of Time, and one Third is quicker than any Man can think or ſpeak diſtinctly, and 227 times quicker than the Motion of Light. For, ſuppoſe the mean Diſtance of the Sun, according to Mr. *Caſſini*, to be 8800000 Miles, and Light to be 10 Minutes in its Paſſage from the Sun to the Earth, the Thirds are 36000, by which divide the Diſtance, it gives 244 Miles in a Third of Time, by which divide 555555 and it gives 227 the Difference.

18. And then 'tis probable there are other Stars, as many or more Millions of Miles beyond the Dog-ſtar, as there are Miles between the Earth and the Dog-ſtar, which muſt make the Motion to be many Millions of Miles in a Third of Time, which, as to all human Conception, is really impoſſible, becauſe it muſt be Millions of Times quicker than Thought.

19. And it is the ſame Thing whether the Earth turns round or the Heavens, as to any Danger of their being flung off the Surface by the rapid Motion,

tion, for the ſame Part of the Heavens as is over the Head at twelve a Clock in the Day, will be directly under the Feet at twelve at Night.

20. As for the annual Motion of the Earth round the Sun, the apparent Stations and Retrogradations of the Planets plainly evince it, ſince theſe irregular Motions cannot be accounted for by Cycles and Epicycles; for if they mov'd in Cycles, ſome Part of their Motion would be directly contrary to their proper Motion in their Revolution, and conſequently muſt loſe all Motion, which was the Cauſe of inventing ſolid Spheres, the Thought of which confutes its ſelf; and tho' the Moon every Month is carried round the Earth, and in its annual Revolution round the Sun, yet it has no Motion contrary to itſelf, as it muſt neceſſarily have, if it were either ſtationary or retrograde.

21. There are beſides the fix'd Stars, which are known in the Heavens by their Twinkling, and always keeping the ſame Situation with reſpect to one another; alſo ſeven Planets which continually alter their Situation; theſe are *Saturn*, *Jupiter*, with their Satellites; *Mars*, the *Earth* with its Satellite the *Moon*, *Venus*, and *Mercury*, which in

their annual Revolution move from the Weſt to the Eaſt, tho' in their diurnal they move from the Eaſt to the Weſt; and thus are the Motions of the Heavens perform'd in Circles, which is the principal Figure in Mathematicks.

22. Hence, to know the chief Properties of it, is the moſt neceſſary Point in Mathematicks, eſpecially the Proportion of the Diameter to the Circumference; for in Aſtronomy the Diameters can only be attained as to any known Diſtance in Miles: In ſhort, all practical Mathematicks depend upon the Circle, as *Trigonometry*, *Navigation*, &c. as it is the Meaſure of all Angles: The grand Principles of Motion are circular, and all the heavenly Bodies, as to appearance, are round; all ſtrait Lines are properly only Projections from Circles, ſuch as Sines, Tangents, Secants, Radii, Chords, *&c.* for if any Diſtance be meaſured upon the Surface of the Earth, it is an Arch, or Part of the Circumference of a Circle, whoſe Center is that of the Earth; if any Diſtance be taken in a ſtrait Line, it is either a Tangent or Chord; if in a perpendicular Line, it is the Radius or verſed Sine; and it is only the Circumference of a Circle, that is properly diviſible into equal Parts, for

all ſtrait Lines have different Increments or Decrements, as they are Tangents, Secants, *&c.* And tho' the Proportion of the Diameter to the Circumference, cannot preciſely be expreſſed in whole or even Numbers, yet it may be ſhewn in Lines, which is the Nature of Surds or incommenſurable Lines; the Difference between the Sides and Diagonal of a Square, cannot be expreſs'd in whole Numbers, tho' the Lines are viſible.

23. There is in all Circles an infinite Surd, in Proportion of the Diameter to the Circumference, as will appear in the Demonſtration. To begin therefore with the Squaring of the Circle, or finding the Length of the Circumference from the Diameter.

CHAP. II.

Squaring of the CIRCLE.

ACcording to the Method of *Euclid*, I ſhall lay down the Propoſition to be proved, tho' 'tis only a Deduction from the Demonſtration.

1. The Circumference of a Circle is equal to three Diameters + $\frac{3}{4}$ the Dif-

ference between the Diameter and the Side of an Equilateral Triangle inſcrib'd + $\frac{3}{4}$ of an infinite Surd; or equal to 3 *da* + $\frac{3}{4}$ *dq* + $\frac{3}{4}$ *qs* + $\frac{3}{4}$ the little infinite Differences between *qu* and *ſu*.

2. And in order to demonſtrate this Propoſition (*Fig.* 1. *P.* 1.) inſcribe an Hexagon *a b c d p*, an equilateral Triangle *a c p*, in a Circle, by 15. *Euc.* 4. By the ſame, the Side of the Hexagon is equal to the Radius.

3. Hence ſix Radius's, or three Diameters, are leſs than the Circumference; and the Difference between them, may be found by the infinitely decreaſing Differences of the Sides of inſcrib'd Polygons; make therefore *a b* equal to *a f*, the Side of a Dodecagon; and *a o* equal to *a b*, the Side of a Hexagon; and *a q* equal to *a c*, the Side of an equilateral Triangle.

4. Let the Lines *r q*, *n o*, *g h*, &c. be called firſt Differences, the Lines *e b* and *e f* being ſuppos'd perpendicular from the Center to the Chords *a b* and *a c*, divides them into equal Parts, by 3. *Euc.* 3. *A c d* is a Right-Angle, becauſe in a Semicircle, by 31. *Euc.* 3. make *a b t*, *a f i*, &c. Right-Angles, and let *Q d*, *o t*, *b i*, &c. be called ſecond Differences.

But in order to make the Demonſtration more clear, it will be neceſſary to premiſe two Theorems.

1. (*Fig.* 4.) That a Line infinitely divided into Halves, and one Half continually taken away, all the remaining Halves, after the firſt Diviſion, are equal to half the Line; as ſuppoſe, from the Line *a b* be taken half, as *a e*, then *e b* remains equal to *e a*; and from *e b* take half, as *e c*, and *c b* remains equal to *c e*, &c. *ad infinitum.* 'Tis plain, here is as much remains as is taken away, and conſequently the Remainders are equal to half the Line. Similar Surfaces decreaſe in a duplicate Proportion, or as Squares; and ſimilar Solids, in a triplicate Proportion, or as Cubes, by dividing their Sides into two equal Parts, by 4. *Euc.* 2, and 19, 6. and 18, 12.

2. (*Fig.* 2.) In the Rectangle Triangle *e a f*, make the Side *a f* double to *a c*, and *a g* equal to *a e*, *a d* equal to *a b*; the Difference *f g*, between *a e* and *a f*, is double to the Difference *c d* between *a b* and *a c*. For the Triangle *a c b* is ſimilar to *a f e*, the Angles at *c* and *f* being right, and *a* common by 4. *Euc.* 6. Hence *a f*: *a c*:: *a e*: *a b*; but *a f* is double to *a c* by ſuppoſition, conſequently *a e* is double to *a b*; but *a e*

is equal to *ag*, by Conſtruction, as likewiſe *ab* equal to *ad*; wherefore *ag* is double to *ad*; take away *af* from *ag*, and *ac* from *ad*, the Remainder *fg* is double to *cd*, by Axiom 20. *Euc.* 1.

5. But now to demonſtrate how a Line may be found equal to the Circumference, by the infinitely decreaſing Lines of the firſt and ſecond Differences: For, ſuppoſe an infinite Number of Polygons inſcrib'd in a Circle, they will terminate in a mathematical Point; for dividing the Sides of Polygons, and continually taking more than half, it will at laſt leave a Point leſs than any aſſignable one, by 1. *Euc.* 10. and 2. 12. Hence the laſt Polygon is equal to the Circle, becauſe each conſiſts of an equal Number of mathematical Points; the Differences likewiſe will be infinite, as they ariſe from the Differences of the Sides of the Polygons; as *ab*, the Side of a Hexagon, is leſs than *af* + *fb*, the two Sides of a Dodecagon, by 2 *gh*; and *ac* the Side of an equilateral Triangle, is leſs than *ab* + *bc*, the two Sides of a Hexagon, by 2 *no*; and *ac*, or *aq* the Side of the ſame equilateral Triangle, is leſs than the Diameter, or *ad*, by *qd*: And thus if the firſt and ſecond Differences are taken backward, they

will infinitely decreaſe; the firſt Differences *r q*, *n o*, *g h*, &c. and the ſecond *q d*, *o t*, *h i*, &c. compared together in a double Proportion, as they are the Homologous Sides of ſimilar Triangles, infinitely decreaſing in a duplicate Proportion: Hence one half the firſt Difference is equal to all the other Differences, by Theorem 1. and 19. *Euc.* 6. for Triangles are in Proportion to one another, as the Sides are.

6. The Triangle *r c d*, is ſimilar to *n b t*, for *n b t* is equal to *n a b*, being both Complements to a Rectangle, equal to *r a c*, being upon equal Arches, which is equal to *r c d*, being likewiſe equal Complements to a Rectangle, by Axom. 3. *Euc.* 1. The Angles at *n* and *r* right, and conſequently the Triangles are ſimilar; again *n b o* and *r c q* are ſimilar, for the Angles at *o* and *q* are equal, being the Angles of *Iſoſceles* Triangles, whoſe vertical Angles at *a*, are equal. Hence the Triangles are ſimilar, and *q c d* and *o b t* are likewiſe ſimilar, as having all their Angles equal.

7. In the next Triangle *f g i*, as the Angle *b a n* is double to *f a g*, being on a double Arch, the Angle *g f i* is equal to half *n b t*, for *g f i* and *g a f* are equal,

being each Complements to a Rectangle; and if the Angle fag had been equal to ban, the Angle gfi would have been juſt equal to nbt; but not to multiply Lines without Neceſſity, double the Angle gfi, which Suppoſition is allowable, becauſe half the Difference in no is for ab, and half for bc; and it is the ſame in gh, for af and fb; for the decreaſing Differences are taken from no for the whole Arch abc; let it therefore be βfi making $f\beta i$ a Right-Angle: Hence nbt and βfi are ſimilar, becauſe all their Angles are equal.

8. From the Similarity of theſe Triangles, their Homologous Sides are in Proportion, or $dq : qr :: ot : on :: ig : g\beta$, and thus they will be *ad infinitum*, and decreaſe in a double Proportion, for dq the ſecond Difference in the firſt Triangle, is double to no the firſt Difference in the ſecond Triangle; for $aq = ac$ is double to an, by 3. *Euc.* 3. and ad is double to $ao = ab$. Hence qd is double to no by Theor. 2. ot is likewiſe a Multiple of gh; and if the Angles gfi and nbt had been equal, it would have been juſt double, as is plain from the former.

9. The firſt Difference rq, &c. is not double to no, &c. for the Sides ar and

aq are not double to an and ao, and the ſecond Differences qd, &c. are ſomething leſs than double to ot, for the ſame Reaſon, by Theo. 2; but the firſt are double to the ſecond, and $qd : qr :: to : on$. Hence here is a little Difference between rq and qd, no and ot, &c. which decreaſe the ſame as they do, by 19. *Euc.* 5. And theſe little Differences, ſuppoſe them infinitely divided, as the firſt Differences are, will decreaſe the ſame as the firſt Differences, and conſequently will determine the Exceſs and Defects in the firſt and ſecond Differences.

10. Hence rs or qd being equal to $2\ no$ and nx, or to equal to twice the next Difference, by Sect. 8. but nx is more than no by the Difference ox; and the ſame happens in all the other little infinite Differences, as between qu and $ſu$, oi and xi; hence the ſame is to be taken in them as in the firſt Difference; wherefore $qd + qs +$ the little infinite Differences between qu and $ſu$, are an Equimultiple of all the Differences decreaſing in a double Proportion. Hence $\frac{1}{2}\,no + \frac{1}{2}\,xo + \frac{1}{2}$ the little infinite Differences between oi and xi, are equal to all the Differences of the Arch abc, and three times that Line equal to all the Differences of the Circle; but 'tis

difficult to find that Line, unleſs it be on the Diameter; wherefore $\frac{1}{4}\,qd + \frac{1}{4}\,qs + \frac{1}{4}$ the little infinite Differences between qu and $ſu$, being equal, will determine the ſame; that is, ab and bc, being equal to one Diameter, three Diameters $+ \frac{3}{4}\,qd + \frac{3}{4}\,qs + \frac{3}{4}$ the little infinite Differences between qu and $ſu$ are equal to the Circle.

11. But to find the Line $qd + qs$, acd being a right-angled Triangle, by 31. *Euc.* 3. the Square of the Diameter ad is equal to the Squares of $ac + cd$. Square therefore the Diameter ad, and half the Diameter cd, ſubtract the latter from the former, and extract the ſquare Root, which Root ſubtract from the Diameter, and the Remainder is qd; which ſubtract from half the Radius or dr (dr being equal to re by Cor. 3. 12. *Euc.* 13.) and the Remainder is qr, which ſubtract from qd, and there remains qs.

12. To give one Inſtance of it in Numbers, let the Diameter be 16 or any other Number.

16 8
16 8
96 64 ½ D. Sq.
16
D. Sq. 256
64
192(13, 85 +
1
23) 092
69
268) 2300
2765)15600
1775

Aq, or the Side of the equilateral Triangle, is 13 ,85 + which ſubtracted from the Diameter, or 16, leaves 2 ,15, the 2 is equal to ½ dr and ,15 = ½ qs: Hence qs is equal to ,30, for the greateſt Side is equal to ½ the Sum, and ½ the Difference, which added to qd, is = 2 ,45; ¼ of which is ½, 11, and three times that Number is ½, 33 = 1 ½ ,33; the other little Differences are found by the Proportion of the two firſt Differences $nq : qd$; but this being near enough, the Curious may ſearch farther into it, which at the moſt can make it but 1 $\frac{1}{2}$, ,42 being = ,$\frac{3}{10}$ of ,$\frac{3}{10}$, &c. wherefore 3 Diameters are equal to 48, which added to the other Number, makes 49 $\frac{1}{2}$,42, which is ſomething leſs than what the common Proportion of 7 : 22 admits of; which is 50 $\frac{2}{7}$: Hence 16 : 50 or 8 : 25 is a nearer Proportion than 7 : 22, or 9 : 28 is the neareſt 10 : 31 being too little. So much for the Squaring of the Circle, or finding the Length of the Circumference.

CHAP. III.

Reflections on the ANOMALIES.

AND now to proceed, and give ſome general Uſes of the Circle.

1. The common Opinion of modern Aſtronomers is, that the Motions of the Planets are in Ellipſis's, and of Comets in Parabolas; and the Reaſon they generally give is, becauſe their viſible Irregularities cannot ſo well be accounted for by the Circle, as by the Ellipſis and Parabola; but why the Laws of Motion ſhould be perform'd in thoſe irregular and unnatural Figures, they

ſeem wholly to paſs by in Silence: And why muſt we erect ſuch an imperfect Machinery in the Heavens, which we find will not anſwer in any Motions here on Earth? An Ellipſis is but an imperfect Circle, with an Infinity of Centres; and why muſt the Creation be robb'd of the moſt beautiful and perfect Motions, in order to make it anſwer the imperfect Mechaniſms of our weak Imagination? For there are no Obſervations ſo exact as to demonſtrate theſe apparent Irregularities, any more by the Ellipſis and Parabola, than by a Circle, if the Nature of an eccentrick Motion be rightly conſidered.

2. For, let there be two equal Circles with different Centers, a Body that evenly revolves in one Circle, ſeen from the Center of the other, will have two Places, a real and apparent one, which is called the Anomaly or irregular Motion; and if the greateſt Anomaly be given, 'tis eaſy to find it, to every Degree of the Quadrant, by dividing it by 90.

3. Let therefore (*Fig.* 1. *Pl.* 3.) $zchi$ and $xhni$ be the two Circles, e and f their Centers; imagine a Planet by an equal Motion, moving in the Circle $zhki$, or round the Center e, let the

Spectator be at the Center of the other Circle, or at *f*; the Lines *e z*, *eh*, *ek*, *e i*, which determine the true Place of the Motion are equal, being Radii of the ſame Circle; the Lines *fz*, *f h*, *fk*, *fi*, which determine the apparent Place, or are the Lines of Viſion are longer or ſhorter, according to the Part of the Circle the Planet is in, *f e* being ſuppos'd the Diſtance of the Centers, *fhe* is the Angle of Anomaly, or equal to the Equation, becauſe vertical to it.

4. Let therefore *fz* be the Apogee, and *fk* the Perigee of the Motion, which are the longeſt and ſhorteſt Lines of Viſion, by 7. *Euc.* 3. ſuppoſe the Planet to move from *z* to *h*, the Line of Motion is before the Line of Viſion, as *e s* is before *f t*, which Diſtance will increaſe till the Planet comes to *h*, the mean Diſtance of the Planet from the Center of the Spectator. For as the Line of Viſion ſhortens, the Angle of Equation increaſes by 21. *Euc.* 1. and here the Anomaly is equal to the whole Equation; but from *h* to *k*, tho' the Line of Motion is before the Line of Viſion, yet the Equation decreaſes, as it increas'd in the firſt Quarter. And when the Planet is come to *k*, the two Lines coincide; and hence no Equation, but as

the Planet goes from *k* to *i*, the Line of Viſion, *or f t* is before the Line of Motion, or *e s*, and the Diſtance will increaſe, tho' the Equation recedes or decreaſes, as it decreas'd in the ſecond Quarter, till it comes to *i* the mean Diſtance, or where the whole Equation is ; and from *i* to *z*, in the fourth Quarter, the Increments of Equation are the ſame as the Increments of the firſt Quarter.

5. But tho' the Equation increaſes in the firſt Quadrant, and decreaſes in the Second, and proceeds or decreaſes in the Third, and recedes or increaſes in the Fourth, the apparent Velocity increaſes in the firſt and ſecond Quadrants, and decreaſes in the Third and Fourth, or increaſes in the Semicircle *z b k*, and decreaſes in *k i z*. For as the Line of Viſion ſhortens, the apparent Velocity increaſes, but as that lengthens that decreaſes: For a Body that moves nearer the Center of Viſion than another, with an equal Velocity, the apparent Velocity is greater in Proportion to their Diſtances from the Center: For inſtance, let (*Fig.* 3. *Pl.* 3.) *e d g* be one Circle, and *b b f* the other, take *b b* equal to *e d*, 'tis plain *b b* is greater, with reſpect to the Circle *b b f*, than *e d* is of the Circle *e d g*; as may be inferr'd from

Cor. 33. *Euc.* 6. Wherefore the apparent Velocity in the Semicircle *zhk*, will increaſe twice the Equation, and decreaſe the ſame in *k i z*, and the even revolving Body will be twice the Equation longer in the upper Part of the Circle, or *hzi* than in *h k i*; for *hn i* is equal to *h z i* by Conſtruction : And ſuppoſe *hni* were mov'd to *h k i*, the two Ends of *hni* would be 2 *nk*, or twice the Equation longer than *h k i*. Here the Lines, and not Surfaces, are conſidered, or ſuppoſe a Body to move round the outer Circle *z*, 1, 2, 3, 4, and another round *zhki*, both with an equal Motion, the inner Body would gain twice the Equation in its whole Revolution of the outer, becauſe the Diameter is ſo much ſhorter ; for *k h*, which is equal to twice the Equation, is the Difference of the Diameters, and the Circumferences are as the Diameters, as will be prov'd afterwards.

Hence if the Eccentricity or Anomaly of the Sun or Planets is known in Miles, 'tis eaſy to find how many Miles the Sun is diſtant from us : For inſtance, to know the mean Diſtance of the Sun, the Triangle *f he* is an Iſoſceles, and the Angle at *h* equal to two Degrees, the other two Angles are 89 ; hence the Side *ef* being given, 'tis eaſy to find by Trigonometry, the other Sides, and *e f* being added or ſubtracted, will give the greateſt and leaſt Diſtances.

6. Now to make this Scheme practically uſeful, and to begin with the Equation of the Sun, which according to the Tables, is about two Degrees when greateſt, which is in the Summer and Winter Solſtice, *Cancer* and *Capricorn*, into which the Sun enters about the firſt of *June*, and the tenth of *December*, conſequently the Lines of Motion and Viſion will coincide in *Aries* and *Libra*, or about the tenth of *March* and the twelfth of *September*.

7. Let *z* repreſent *Aries* or *March*, *h Cancer* or *June*, *k Libra* or *September*, *i Capricorn* or *December*; this Scheme is according to the Tables of Equation, which if true, Aſtronomers are guilty of an Error, in making the greateſt and leaſt Diſtances of the Sun to be near the greateſt Equation, when it muſt be 90 Degrees diſtant from it; for the greateſt and leaſt Diſtances are in *z* and *k*, or *March* and *September*, as is plain from the Scheme, Tables of Equation, and 7. *Euc.* 3.

8. To proceed, Suppoſe the Sun be in *z*, or the tenth of *March*, when ac-

cording to the Tables, there is no Equation in moving from thence to *h*, or *Cancer*, the Line *e s* of the Sun is before the Line *ſ t* of Viſion, and at *h*, or the tenth of *June*, the Diſtance or Angle is equal to two Degrees, or the whole Equation, which in Time is equal to eight Minutes; but when it moves from *h* to *k*, or the tenth of *June*, to about the twelfth of *September*, it will loſe two Degrees or eight Minutes in Equation, in the third Quarter; or from *k* the twelfth of *September*, to *i* the tenth of *December*, as the Line of Viſion is before the Line of the Sun, the Equation recedes or decreaſes in the fourth and laſt Quarter, from the tenth of *December* to the tenth of *March*, the Equation will increaſe or proceed, as it decreas'd the laſt Quarter.

9. But then, beſides this Increaſe and Decreaſe in Equation, the Sun will likewiſe gain in Velocity in the firſt Semicircle, or from the tenth of *March* to the twelfth of *September* four Degrees, becauſe the Line of Viſion becomes ſo much ſhorter; and in the ſecond Semicircle, or from the twelfth of *September* to the tenth of *March*, it looſes four Degrees in Velocity, as the Line of Viſion lengthens ſo much by Sect. 5:

10. Hence, beſides the daily Equation of the Sun, there is likewiſe what may properly be called an annual Equation; for its Motion, from the tenth of *June* to the tenth of *December*, or from *Cancer* to *Capricorn*, is performed in four Days fifteen Minutes leſs than from *December* to *June*, or from *Capricorn* to *Cancer*; for the Motion of the Sun is 59 Primes 8 Seconds a Day, and the Part of the Circle *hki*, is four Degrees leſs than *izh*, which is equal to four Days fifteen Minutes; and then the Sun is twice the Equation farther from the Earth in *Aries* or *March*, than in *Libra* or *September*, as is plain from the Scheme; for *fz* the Sun's greateſt Diſtance, and *fb* are equal; but *fb* is twice the Equation longer than *fk*, the Sun's leaſt Diſtance.

11. Hence likewiſe may a Reaſon be given why Dials are ſometimes faſter and ſometimes ſlower in the Morning than in the Afternoon, which Difference is eaſily obſerv'd in true Dials, by a good Clock or Watch; for when the Line of the Sun is before the Line of Viſion, as from *Aries* to *Libra*, the Dial will be too faſt in the Morning, and too ſlow in the Afternoon; and from *Libra* to *Aries*, the Line of Viſion is before

the

the Line of the Sun; and the Dials are too ſlow in the Morning, and too faſt in the Afternoon, according to the Tables of Equation, eight Minutes being the greateſt; and by this Dial and Watch may the true Equation of the Sun be determined.

12. A Pendulum may eaſily be adjuſted, to ſwing the Equation: For ſuppoſe the whole Pendulum to be the Radius or Sine of 90, take the Sine of 4 for the ſhorteſt, reckoning from the Center, as it is in the Scheme, or make the Pendulum to lengthen and ſhorten, according to the Increment or Decrement of the Line of Viſion, and its Motion if regular, will be juſt as the Sun's; For inſtance, from the tenth of *March* to the twelfth of *September*, ſhorten the Pendulum gradually to the Sine of 4, which lengthen again from *September* to *March*; and this is the only Way to accommodate any Motion with the Sun: For to found an Equation on the Difference of the Equator and Ecliptick muſt evidently be falſe, ſince they are both great Circles of a Sphere, and conſequently equal, only the Degrees of the Ecliptick are greater or leſſer, as it is nearer or farther from the Pole, otherwiſe the Squares of the four Quar-

ters of the Ecliptick are equal to the Squares of the four Quarters of the Equator, together with four Times the Squares of the whole Declination, and conſequently not equal Circles, which is abſurd. So much for the Equation of Time. And if the Tables are true, the Demonſtration is infallible. Here are inſerted the Tables of Equation and Velocity, which as they are really different, becauſe the Equation differs every Quarter, but the Velocity every half Year, there are two diſtinct Tables of them.

TABLES of EQUATION, made from the Aſtronomical TABLES, and from the greateſt Diſtance of the Sun to every ſixth Day.

The Sun faſter than equal Time, to be added.			*The Sun ſlower than equal Time, the Remainder to be added.*		
Days	M.	S.	Days	M.	S.
Mar. 10 *Aries*		30	*June* 15	7	30
16	1	00	21	7	00
22	1	30	27	6	30
28	2	00	*July* 3	6	00
April 3	2	30	9	5	30
9 *Taurus*	3	00	*Leo* 12		
15	3	30	15	5	00
21	4	00	21	4	30
27	4	30	27	4	00
May 3	5	00	*Auguſt* 2	3	30
9	5	30	8	3	00
10 *Gemini*			*Virgo* 12		
15	6	00	14	2	30
21	6	30	20	2	00
27	7	00	26	1	30
June 3	7	30	*Sept.* 2	1	00
9	8	00	8	0	00
11 *Cancer*			*Libra* 12		

TABLES

TABLES of EQUATION, &c.

The Sun ſlower than equal Time, to be ſubtracted.

Days	M.	S.
Sept. 14	0	30
20	1	00
26	1	30
Octob. 2	2	00
8	2	30
Scorp. 12		
14	3	00
20	3	30
26	4	00
Novem. 1	4	30
7	5	00
Sagit. 11		
13	5	30
19	6	00
25	6	30
Decem. 1	7	00
7	7	30
Capri. 11		
13	8	00

The Sun faſter than equal Time, the Remainder to be ſubtracted.

Days	M.	S.
Decem. 19	7	30
25	7	00
31	6	30
Januar. 6	6	00
Aqua. 9		
12	5	30
18	5	00
24	4	30
30	4	00
Feb. 5	3	30
Piſces 8		
11	3	00
17	2	30
23	2	00
29	1	30
March 6	1	00
10	0	00

TABLES of VELOCITY, which is likewiſe made from the Sun's greateſt Diſtance, *Sect.* 12. *Chap.* 3. made to every 3 Degrees.

The Sun faſter than equal Motion to be added.				*The Sun faſter than equal Motion.*			
Signs	deg.	d.	p.	Signs	deg.	d.	p.
Aries	3	0	4	*Cancer*	3	2	4
	6	0	8		6	2	8
	9	0	12		9	2	12
	12	0	16		12	2	16
	15	0	20		15	2	20
	18	0	24		18	2	24
	21	0	28		21	2	28
	24	0	32		24	2	32
	27	0	36		27	2	36
	30	0	40		30	2	40
Taurus	3	0	44	***Leo***	3	2	44
	6	0	48		6	2	48
	9	0	52		9	2	52
	12	0	56		12	2	56
	15	1	00		15	3	0
	18	1	4		18	3	4
	21	1	8		21	3	8
	24	1	12		24	3	12
	27	1	16		27	3	16
	30	1	20		30	3	20
Gemini	3	1	24	*Virgo*	3	3	24
	6	1	28		6	3	28
	9	1	32		9	3	32
	12	1	36		12	3	36
	15	1	40		15	3	40
	18	1	44		18	3	44
	21	1	48		21	3	48
	24	1	52		24	3	52
	27	1	56		27	3	56
	30	2	00		30	4	00

TABLES of VELOCITY, &c.

The Sun ſlower than equal Motion, the Remainder to be added.

Signs	deg.	d.	p.
Libra	3	3	56
	6	3	52
	9	3	48
	12	3	44
	15	3	40
	18	3	36
	21	3	32
	24	3	28
	27	3	24
	30	3	20
Scorpio	3	3	16
	6	3	12
	9	3	8
	12	3	4
	15	3	0
	18	2	56
	21	2	52
	24	2	48
	27	2	44
	30	2	40
Sagitt.	3	2	36
	6	2	32
	9	2	28
	12	2	24
	15	2	20
	18	2	16
	21	2	12
	24	2	8
	27	2	4
	30	2	0

The Sun ſlower than equal Motion.

Signs	deg.	d.	p.
Capric.	3	1	56
	6	1	52
	9	1	48
	12	1	44
	15	1	40
	18	1	36
	21	1	32
	24	1	28
	27	1	24
	30	1	20
Aquarius	3	1	16
	6	1	12
	9	1	8
	12	1	4
	15	1	0
	18	0	56
	21	0	52
	24	0	48
	27	0	44
	30	0	40
Piſces	3	0	36
	6	0	32
	9	0	28
	12	0	24
	15	0	20
	18	0	16
	21	0	12
	24	0	8
	27	0	4
	30	0	0

13. But when the Equation is greater there is a greater Difference, as in the Moon, whoſe greateſt Equation, according to the Tables, is about five Degrees, and greateſt likewiſe in *Cancer* and *Capricorn*; and the Line of Viſion and of the Moon, coincide in *Aries* and *Libra*: Hence, according to what has been ſaid of the Sun, the Moon gains five Degrees in Equation from *Aries* to *Cancer*, which it loſes from *Cancer* to *Libra*, and loſes or recedes five Degrees from *Libra* to *Capricorn*, which it gains from *Capricorn* to *Aries*.

14. And, beſides the Equation, it will likewiſe gain in Velocity ten Degrees from *Aries* to *Libra*, which it loſes from *Libra* to *Aries*; and the Moon is twice the Equation nearer the Earth in *Libra* than in *Aries*.

15. The under Part of the Circle being ten Degrees leſs than the upper, and the Motion of the Moon about thirteen Degrees in twenty four Hours, its Stay is about nineteen Hours leſs from *Cancer* to *Capricorn*, than from *Capricorn* to *Cancer*, which are ſome of the Moon's Irregularities; the other will be ſhewn in the Calculation of the Moon's Place: Here are inſerted the TABLES of the Moon's Equation and Velocity,

Velocity which are of the ſame Nature as thoſe of the Sun, and for want of diſtinguiſhing them, has caus'd the Difficulties in finding the Moon's Place; the Uſe of them will be ſhew'd in calculating the Moon.

TABLES of the Moon's Equation and Velocity, taken from the greateſt Diſtance to every three Degrees.

A TABLE of EQUATION.

The Moon faſter to be added.				*The Moon ſlower, the Remainder to be added.*			
Signs	deg.	d.	p.	Signs	deg.	d.	p.
Aries	3	0	10	*Cancer*	3	4	50
	6	0	20		6	4	40
	9	0	30		9	4	30
	12	0	40		12	4	20
	15	0	50		15	4	10
	18	1	0		18	4	0
	21	1	10		21	3	50
	24	1	20		24	3	40
	27	1	30		27	3	30
	30	1	40		30	3	20
Taurus	3	1	50	*Leo*	3	3	10
	6	2	0		6	3	0
	9	2	10		9	2	50
	12	2	20		12	2	40
	15	2	30		15	2	30
	18	2	40		18	2	20
	21	2	50		21	2	10
	24	3	0		24	2	0
	27	3	10		27	1	50
	30	3	20		30	1	40
Gemini	3	3	30	*Virgo*	3	1	30
	6	3	40		6	1	20
	9	3	50		9	1	10
	12	4	0		12	1	0
	15	4	10		15	0	50
	18	4	20		18	0	40
	21	4	30		21	0	30
	24	4	40		24	0	20
	27	4	50		27	0	10
	30	5	0		30	0	0

TABLES of the Moon's Equation, &c.

The Moon ſlower to be ſubtracted.				*The Moon faſter, the Remainder to be ſubtracted.*			
Signs	deg.	d.	p.	Signs	deg.	d.	p.
Libra	3	0	10	*Capric.*	3	4	50
	6	0	20		6	4	40
	9	0	30		9	4	30
	12	0	40		12	4	20
	15	0	50		15	4	10
	18	1	0		18	4	0
	21	1	10		21	3	50
	24	1	20		24	3	40
	27	1	30		27	3	30
	30	1	40		30	3	20
Scorpio	3	1	50	*Aquarius*	3	3	10
	6	2	0		6	3	0
	9	2	10		9	2	50
	12	2	20		12	2	40
	15	2	30		15	2	30
	18	2	40		18	2	20
	21	2	50		21	2	10
	24	3	0		24	2	0
	27	3	10		27	1	50
	30	3	20		30	1	40
Sagitt.	3	3	30	*Piſces*	3	1	30
	6	3	40		6	1	20
	9	3	50		9	1	10
	12	4	0		12	1	0
	15	4	10		15	0	50
	18	4	20		18	0	40
	21	4	30		21	0	30
	24	4	40		24	0	20
	27	4	50		27	0	10
	30	5	0		30	0	0

A TABLE of the Moon's Velocity.

The Moon faſter to be added.				*The Moon faſter to be added.*			
Signs	Deg.	d.	p.	Signs	Deg.	d.	p.
Aries	3	0	10	*Cancer*	3	5	10
	6	0	20		6	5	20
	9	0	30		9	5	30
	12	0	40		12	5	40
	15	0	50		15	5	50
	18	1	0		18	6	0
	21	1	10		21	6	10
	24	1	20		24	6	20
	27	1	30		27	6	30
	30	1	40		30	6	40
Taurus	3	1	50	*Leo*	3	6	50
	6	2	0		6	7	0
	9	2	10		9	7	10
	12	2	20		12	7	20
	15	2	30		15	7	30
	18	2	40		18	7	40
	21	2	50		21	7	50
	24	3	0		24	8	0
	27	3	10		27	8	10
	30	3	20		30	8	20
Gemini	3	3	30	*Virgo*	3	8	30
	6	3	40		6	8	40
	9	3	50		9	8	50
	12	4	0		12	9	0
	15	4	10		15	9	10
	18	4	20		18	9	20
	21	4	30		21	9	30
	24	4	40		24	9	40
	27	4	50		27	9	50
	30	5	0		30	10	0

A TABLE of the Moon's Velocity.

The Moon ſlower, the Remainder to be added.				*The Moon ſlower to be added.*			
Signs	Deg.	d.	p.	Signs	Deg.	d.	p.
Libra	3	9	50	*Capricorn*	3	4	50
	6	9	40		6	4	40
	9	9	30		9	4	30
	12	9	20		12	4	20
	15	9	10		15	4	10
	18	9	0		18	4	0
	21	8	50		21	3	50
	24	8	40		24	3	40
	27	8	30		27	3	30
	30	8	20		30	3	20
Scorpio	3	8	10	*Aquarius*	3	3	10
	6	8	0		6	3	0
	9	7	50		9	2	50
	12	7	40		12	2	40
	15	7	30		15	2	30
	18	7	20		18	2	20
	21	7	10		21	2	10
	24	7	0		24	2	0
	27	6	50		27	1	50
	30	6	40		30	1	40
Sagittary	3	6	30	*Piſces*	3	1	30
	6	6	20		6	1	20
	9	6	10		9	1	10
	12	6	0		12	1	0
	15	5	50		15	0	50
	18	5	40		18	0	40
	21	5	30		21	0	30
	24	5	20		24	0	20
	27	5	10		27	0	10
	30	5	0		30	0	0

16. But if the Equation be much greater, as in Comets, which ſuppoſe to be 80 Degrees, the Comet from *Aries* to *Cancer*, (if the greateſt Diſtance is in *Aries*) gains in Equation 80 Degrees, which is 53 Primes for every Degree, which it looſes from *Cancer* to *Libra*; and from *Libra* to *Capricorn* it looſes or recedes 80 Degrees, which it gains from *Capricorn* to *Aries*; and then it gains 160 Degrees in Velocity from *Aries* to *Libra*, which it looſes from *Libra* to *Aries*; and the under Part of the Circle being 160 Degrees leſs than the upper one, the Stay of the Comet in that Part is as 340:20 or 17:1, and the Comet is twice the Equation, or 160 Degrees farther in *Aries* than in *Libra*.

17. Then, beſides the finding theſe Irregularities from the Equation, 'tis eaſy to find the Equation from two or three Obſervations of the Irregularities: For ſuppoſe the Sun gains or loſes five Seconds of Time in twenty four Hours, multiply it by 90, and it gives two Degrees Equation. So much for Anomalies.

CHAP. IV.

A general Scheme for DIALLING.

FROM the Circle likewiſe may the whole Myſtery of practical Dialling be diſcovered, by two concentrical Circles.

1. Draw the Circle (*Fig.* 2. *Pl.* 3.) *luda*, which ſuppoſe to repreſent the Latitude; let its Center *p*, be the Center of the Dial, if it has a Center, divide the Quadrant *du* into Degrees, and take 51° 30′, or any other Latitude, that the Dial is made for, from *d* to *f*, which is the Height of the Gnomen; then take off the ſame Latitude on the other Side, and with a Ruler mark the Point *g*, which is the Sine of the Latitude, or equal to the Diſtance of the Equator from the Latitude of the Place, which mark from *d* to *q*, and draw the Circle *etqc*, which will repreſent the Equator, and may be divided into twenty four equal Parts, for Hours 48 for half Hours, or any other Diviſions; then put *dg* or the Latitude from *P* to *h*, for the Height of the Pole is always equal to the Latitude of the Place; here the leſſer Sines are taken for the Diſtance

of the Equator and Height of the Pole, and a Ruler laid from *b* to each of the Diviſions on the Equator, and Marks made in the Circle of Latitude, and the Ruler laid again from the Center, or *P*, to each of theſe Marks, will cut off the true Hours on the Equator, *probatum eſt.*

2. This way of dividing Hours will ſerve for any Dials, as *Eaſt* or *Weſt*, by laying the Ruler ſideways, and taking off the Diſtances of the Hours according to the Latitude; as likewiſe reclining and declining, by placing the Gnomen parallel to the Meridian, and the Angle or Height equal to the Latitude.

3. The Figure proves itſelf, for *P b* is equal to the Elevation of the Pole, equal *q d* the Latitude; the Elevation and Latitude being always equal, and the Equator is equally divided, and the Circle of Latitude unequally, according to its Diſtance from the Equator. So much for Dialling.

CHAP.

CHAP. V.

Of the mechanical POWERS.

FROM the Circle likewiſe may be demonſtrated the Myſtery of the mechanical Powers.

1. As there is a mutual Proportion between the Velocity and Power, hence a double Velocity requires but half the Power that a ſingle one does; ſo if a ſingle Velocity moves a Power, a double one will move a double Power; for if *a* is equal to *b*, 2 *a* are equal to 2 *b*, which is the Nature of the mechanick Powers, as their Actions are explicated by the Circle.

2. For Inſtance, let there be two concentrical Circles *alb* and *ghd* (*Fig.* 5. *Pl.* 3.) let the Diameter of the outer Circle be double to the inner one, the Circumference is likewiſe double; for draw the Chords *al* and *gh*, and *ca*: *cg*::*al*:*gh* being ſimilar Triangles, by 4. *Euc.* 6. but *ca* is double to *cg* by Suppoſition: Hence *al* is double to *gh*, and the Arch *la* double to *hg*, by Cor. 3. 33. *Euc.* 6. and if the Quarters are double, the Wholes are, or the Circumference *alb* is double to *ghd*; in Mechanicks

the Circumferences and not Surfaces are conſidered; and hence is demonſtrated the Proportion between the Power and Velocity: For ſuppoſe the Radius, or *c a*, to repreſent the Power and the Circumference, or *a l b* the Velocity; and let the Radius or Power be turn'd round the Center *c*, the Velocity at *a* is double to the Velocity at *g*, as the Powers are till it comes to the Perpendicular, and when paſs'd it is the ſame: For Inſtance, Suppoſe the Radius or Power to be depreſs'd, or rais'd up to *k*, (*Fig.* 6. *Pl.* 3.) the Velocity and Powers retain the ſame Proportion as the Arks, and are double, as *c b* is double to *c d*, for *c k d* and *c f d* are ſimilar Triangles: Hence *c f*:*c k*::*c d*:*c d*, but *c k* is double to *c f*, and conſequently *c d* to *c d*, the Arks or Velocity are the ſame. And thus the Velocity is always in Proportion to the Radius or Power, tho' the real Power, as it raiſes or falls towards the Perpendicular, decreaſes, as the verſed Sines or *d b*, or as the Squares of the Diameter or Length. But the better to illuſtrate the Nature of it, 'twill be neceſſary to give Inſtances in the five mechanick Powers, by only giving the ſimple Powers without conſidering how they are compounded.

1. Suppoſe *ab* the Beam of a Balance, let *c l* be the Handle at the Center, an equal Power in *a* and *b*, are in *Equilibrio*; but ſuppoſe the Power at *b* remov'd to *d*, it requires a double Power in *d* to preſerve the *Equilibrium*, as the Velocity at *b* is double to the Velocity at *d*; and thus it will require more or leſs Power, in Proportion to the different Lengths or Powers of each Side of the Beam.

2. Suppoſe *a b* to be a Lever, let *c* be the Fulcrum, a Power at *b* will raiſe double the Weight that a Power at *d* does, becauſe the Velocity is double.

3. In the Pulley, the Multiplying of Shivers leſſens the Power in a double Proportion, or by halves; for if there are two Shivers, the Chord that moves in them will require twice the Length or Circumference to raiſe the Power, the ſame Height as one Shiver does, and conſequently takes off half the Power; and if a third Shiver be added, it takes off half the remaining Weight, for the Velocity is triple.

4. In the Wedge, the broader it is at the Top, in Proportion to the Radius, the more it widens or extends the Log or Impediments, conſeqnently requires a proportionably greater Power, the

Top and Radius are reciprocal; for if the Top and Radius of one Wedge be double to the Top and Radius of another, they are reciprocally equal in Velocity, and require the ſame Power.

5. In the Screw, the Velocity is in Proportion to the Leaves and Diameter of the Cylinder, as if one Leave is double to another of equal Diameters, the leſſer will turn round twice to the other's once, to raiſe the Power the ſame Height: Hence the Velocity being double, takes off half the Power; but if the Leave and Diameter of one Screw, be double to the Leave and Diameter of the other Screw, they are reciprocally equal.

6. This is the Nature of the Circumference of a Circle when applied to Mechanicks, but if the Circle be conſidered as to its Area or Surface, the Proportion of two Circles one to another are duplicate; or as the Squares of their Diameters, and Solids or Spheres are triplicate; or as the Cubes of their Diameters, by 1. and 18. *Euc.* 12. Hence the Area of a Circle, whoſe Diameter is two Foot, is four times as big as a Circle of one Foot Diameter, or 4 : 1; and a Globe of two Foot Diameter is

eight times as big as a Globe of one Foot Diameter, or 8 : 1.

7. And from the Nature of the Circle, Sir *Iſaac Newton*, and moſt Moderns, hold that Gravity, and all Qualities, decreaſe as the Squares of their Diſtance from the Center; becauſe their Spiſſitude decreaſes as the Area of Circles, which is as the Squares of their Diameters. But this Aſſertion muſt evidently be falſe, for all Bodies that produce theſe Qualities, as Heat, Cold, Smell, &c. diffuſe them all round a Center, like a Sphere or Globe, and the Space they take up is ſpherical; but as the Space increaſes the Spiſſitude of Qualities decreaſes, and the Increment of ſpherical Space is in a triplicate Ratio, or as the Cubes of the Diameters, conſequently the Force of Qualities decreaſe in a triplicate Proportion of their Diſtance from the Center.

8. And tho' Gravity is by ſome reckoned a Quality, yet it is highly improbable, or rather impoſſible, ſince it does not proceed from any Effluvias, as other Qualities do, neither is it eſſential to Matter, as having its tendency every way, which is impoſſible. Hence then it muſt be reſolv'd into an immediate Act of the Divine Power, and

Will; and if God has fixed Centers of Gravity, and the Force of it decreaſe in Proportion to the Diſtance from the Center, it muſt be in a triplicate Ratio, or as the Cubes of the Diameters, becauſe it diffuſes it ſelf after a ſpherical Manner: Hence if any Quality, as Heat, *&c.* has eight Degrees of Intenſneſs at one Foot Diſtance from the Center, at two Foot Diſtance it has but one Degree of Intenſneſs; that is, ſuppoſing it has no hindrance in diffuſing itſelf round the Center, as the Sun and Stars have none; hence the Heat and Light of theſe Bodies muſt decreaſe in a triplicate Ratio, or as the Cubes of their Diſtance; and the ſame may be ſaid of the Denſity of the Atmoſphere, which decreaſes as the Cubes of the Diſtance from the Center of the Earth. So much for Mechanicks and the Digreſſion.

CHAP. VI.

The Diviſion of a MONACHORD.

THE Diviſion of a Monachord, or the Grounds of Muſick demonſtrated from the foregoing Reflections, and the Circle.

1. The Propagation of Sounds is ſpherically, or as Cubes; for the Vibrations of Air are diffus'd after a ſpherical Manner, becauſe diffus'd all round a Center, by *Chap.* 5. *Sect.* 7. This accounts for the whole Myſtery of Muſick.

2. Hence then, as a Globe or Sphere of one Foot Diameter, is eight times leſs than a Globe of two Foot Diameter, or 8 : 1, ſo in a String or Monachord of two Foot Length, the Spiſſitude of the Vibration is eight times leſs than in a String of one Foot Length, or 8 : 1; for the Spiſſitude of Vibrations decreaſing as Cubes, the ſame as the Spiſſitude of other Qualities: Hence an Octave is $\frac{1}{2}$ of a Monachord, or 2 : 1, as the Cube Root of 8 is 2, and the Cube Root of 1 is 1, the other intermediate Notes are the remaining Surds after the Cube Root is extracted, put like a Fraction over the Reſolvend: For Inſtance, a fifth is three Notes from the eight, extract the Cube of 3 there remains 2, which put over 3 the Reſolvend, it is $\frac{2}{3}$ of the whole Monachord, the remaining Surd of a 4th, after the Cube Root is extracted, is $\frac{3}{4}$ of a third $\frac{4}{5}$ of a ſecond or third Minor, $\frac{5}{6}$ of a ſixth is $\frac{5}{6}$ of a ſeventh or ſixth Major is $\frac{6}{7}$, the two laſt Diviſions to be taken backward, becauſe

a ſixth Major and Minor are at the ſame Diſtance from the Octave, as the third Major and Minor are from the ground Note: Thus, ſuppoſing the Diameter of the Circle *gag* be divided according to the Surds, it contains two Octaves; and if the Squares of the periodical Revolutions of the Planets, are as the Cubes of their Diſtances, the Syſtem of the Sun and Planets, are as the Diviſion of the Monachord; for the Diviſions of the Monachord decreaſe as Cube Surds, and Circles are to one another as Squares: Hence the Muſick of the Spheres; and hence, if the Diſtance of any Planet from the Sun, or of two Planets one from another, is given the Form and Diſtance of the whole Syſtem is known; for Inſtance, ſuppoſe the outer Circle to be 1000000, the Diſtances are in this order:

Saturn 800000 $= \frac{5}{6}$
Jupiter 750000 $= \frac{4}{5}$
Mars 666667 $= \frac{3}{4}$
Earth 500000 $= \frac{2}{3}$
Venus 200000 $= \frac{5}{6}$
Mercury 116666 $= \frac{6}{7}$

But how near theſe Diſtances anſwer, I ſhall not pretend to determine; only the Diſtances of the outer Planets are to be taken when in oppoſition, and of the two inner ones when in Conjunction with the Sun.

The Nature of Pendulums are likewiſe in a muſical Proportion; for the Cube Roots of the Lengths are as the Squares of the periodical Times, and the Arks made by the Swings of Pendulums are in the ſame Proportion. For Inſtance, Suppoſe the Radius *g a* of the Circle *g a g* be adapted to any Time to make a Pendulum ſwing a 5th, it is $\frac{2}{3}$ of the Length, *&c.*

3. Hence likewiſe the natural Aſcent of an Octave, according to the true Diviſion of the Monachord, is a Tone Semi-tone, three Tones Semi-Tone Tone, which is a natural flat Key, but the Aſcent may be altered according as the Turns are flat or ſharp. For Inſtance, in a ſharp Tune the firſt Note begins from the 4th or *c*; and then it is two Tones Semi-Tone, two Tones Semi-Tone Tone; or if the firſt Note begins at *d*, or the fifth, the Aſcent is Tone, Semi-Tone, two Tones, Semi-Tone two Tones, which is another flat Key, becauſe the Semi-Tone is the ſecond Note in the Octave. There are ſeven different Keys, and 40320 different Alterations or Changes, including the Octave,

or 5040 in the ſeven original Sounds, there being ſo many Alternations in each of the Numbers; the intermediate Surds or Fractions may be multiplied and divided as vulgar Fractions, for continuing or leſſening their Ratio's. For Inſtance, to continue the Ratio of a Fourth and Fifth, multiply $\frac{2}{3}$ by $\frac{3}{4}$, it is $\frac{6}{12}=\frac{1}{2}$, or an Octave the Difference between them is $\frac{8}{9}$ a Tone.

4. Hence, from the Scheme, 'tis eaſy to learn the *Solfa* on any Key, conſidering that *Mi* is always upon the half Notes, which are two in the Octave between *a* and *b*, *e* and *f*: For Inſtance, in the firſt natural flat Key it is *Sol*, *La*, *Mi*, *Fa*, *Sol*, *La*, *Mi*; but if it is a ſharp Key in *c*, 'tis *Fa*, *Sol*, *La*, *Mi*, *Fa*, *Sol*, *La*, *Mi*; and thus it may be continued infinitely. View the Scheme diligently which unravels the whole Myſtery.

5. And here ſome few Remarks may be made on Poetry; for by this Diviſion of the Monachord, the Nature and Muſick of the Verſes may be eſtimated. An Hexameter Verſe conſiſts of twelve Sounds, containing an Octave, and a Fifth a Pentameter of ten Sounds, an Octave and a greater Third; and if the Spondees and Dactyls are placed according to the Tones and Semi-Tones on

the Monachord, it would not only be more Muſical, but anſwer to the general Rules that are given for ſuch Verſes: For Inſtance, in the Hexameter it would be Spondee, Dactyl, two Spondees, Dactyl, Spondee; in the Pentameter Dactyl, Spondee, half Spondee, two Dactyls, half Spondee; but theſe may be altered as the Keys in the Octave, and this anſwers very near to the Tones and Semitones on the Monachord. *Hudibras*'s Verſes are an Octave; but, for want of Spondees and Dactyls in the *Engliſh* Language, we are forc'd to rhime at the End of the Verſes to give the Jingle to the Poetry. Iambicks may be ſo contriv'd as to have their ſhort Notes anſwer to the Semitones.

6. Hence likewiſe may be demonſtrated why Colours have the ſame Proportion as Sounds have, becauſe Light is propagated after a ſpherical Manner, or as Cubes, as mentioned *Chap.* 5. *Sect.* 7. Suppoſe therefore 1 the Cube to be pure white or contain all the Rays, the other Colours differ as the Cube Roots of the other Numbers, becauſe Colours ariſe from a greater or leſs Spiſſitude of Rays extracting, therefore the Cube Roots; the remaining Surds are the ſame as on the Monachord, and there may be 40320

different

different Colours, including the eighth or 5040 in the ſeven original Colours. Hence we may ſee the Harmony and Agreement that is in the Myſteries of the Chriſtian Religion and Nature, as they agree in the Number 7, as 7 Planets; 7 original Sounds, 7 original Colours; and it muſt be the ſame in Taſte and Smell, becauſe all Qualities are of the ſame Nature as to their Spiſſitude; and then in Religion, the World was created in 7 Days, the 7 Sacraments, the 7 Degrees in Order, the myſtical Propheſies of the Revelations, are in the Number 7; and 'tis highly probable we ſhould have had 7 Senſes, had we not ſin'd in *Adam*, and might have ſeen God and convers'd with Angels. So much for Muſick.

CHAP. VII.

Of REFRACTIONS.

FROM the Circle likewiſe may the Myſtery of Refractions, as well as the Difference of the Corpuſcles which conſtitute Diaphanous Bodies, be diſcovered.

1. Refractions are owing to the little Bendings of the Rays of Light in paſſing through the Diaphanous Body; and the bigger the Corpuſcles of the Body are, the bigger are the Angles of Refraction, as Glaſs and Water have a greater Refraction than Air, and the broader or deeper the Diaphanous Body is, the greater is the Angle; for as the little impenetrable Corpuſcles deflect the Rays from going on in a ſtrait Line, which by Experience is found to be the natural Motion, the firſt Deflection gives the true Angle of Refraction; but if the Ray penetrates farther, it meets with other Corpuſcles which increaſes the Angle of Refraction.

2. For Inſtance (*Fig.* 11. *Pl.* 2.) let *a b c* repreſent a Priſm, and *ſc* a Ray of the Sun falling upon the broadeſt Part, and *ſb* another Ray falling upon the narroweſt Part, the firſt Corpuſcle at *c*, gives the true Angle of Refraction, and the Particle next to that will carry the Ray higher, *&c.* (the Rays always tending to the Perpendicular, becauſe lighteſt.) But where there is but one Particle, as at *f*, it has only one Refraction; this is what anſwers to the common Experiments of Refractions; for in an equiangular Priſm, ſome Rays re-

fract more than others; but that the different Refractions are owing to the different Broadneſs of the Priſm, may be demonſtrated from the Property of a Circle, and the Tables of Refraction.

3. For, ſuppoſe two Circles drawn as *t a* and *e h* (*Fig.* 3. *Pl.* 2.) let *t a* repreſent the Atmoſphere, and *e h* the Earth, let *ſe*, *ſe* be two Rays of the Sun, the one Horizontal and the other Perpendicular, according to the Tables the Horizontal Refraction, or *ſe*, is 33 Primes, and none in the Perpendicular; though this muſt evidently be falſe from Experiments; for a Stick that ſtands Perpendicular refracts no more of one Side than of the other, but looks bigger every way, and has a Refraction round it; the ſame it muſt be in the Air, tho' not ſo viſible. A Star perpendicular looks bigger than when it is oblique; but that the Angle of the Horizontal Refractions are bigger than the perpendicular ones, is owing to the different Broadneſs of the refracting Medium; for *t e* is longer than *e a*, by 7. *Euc.* 3. and conſequently the broader the Medium is, the greater is the Refraction, which proves one Part of the Propoſition.

4. To prove the other; the Difference of the Angles of Refraction ſhew the

different Bigneſs of the Corpuſcles, as the Sine of Refraction out of Air into Glaſs, is 11 : 17. and this gives the Difference of the Corpuſcles, or the Particles of Glaſs are near $\frac{1}{3}$ bigger than thoſe of Air; 'tis probable the irregular Refractions do ariſe from the different Form of the Particles; ſuch as Square, Pentagon, &c. which conſtitute the refracting Medium.

CHAP. VIII.

Of converging RAYS.

FROM the Circle may be ſhewed a Method of finding the Focus of converging Rays.

1. All Rays are found to move in ſtrait Lines, which falling upon the Surface of a Sphere, penetrate it at Right-Angles with the Tangent, and conſequently meet in the Center, which are ſcatter'd in the going out of the Glaſs that is Spherical; but if in a Segment of a Sphere the Rays paſs through, and converge in the Center of the Sphere of which the Segment is a Part, only Allowances muſt be made for the Re-

Refraction which is always toward the Perpendicular.

2. To find where the Rays converge, let the Diameter or Length and the greateſt Thickneſs of a Segment or ſingle Convex Glaſs be given: For inſtance, (*Fig.* 7. *Pl.* 2.) let *e a b* be the Segment or Convex Surface of the Glaſs, divide the Segment into two equal Parts, by a Perpendicular, as *a c* and compleat the Figure, *a e d* and *a c e* are ſimilar Triangles; for *a e c* is a Right-Angle, being a Semicircle, and *a d e* is right by Conſtruction, the Angle at *a* common to both, wherefore *d a* : *a e* : : *a e* : *a c*, that is, the Square of the Chord of half the Length of the Segment is equal to the Rectangle under the greateſt Breadth and Radius of the Circle; and this determines the Focus. Hence, ſquare the half Chord and divide it by the greateſt Breadth, and it gives the Center of the converging Rays.

3. Glaſſes that are made from the Segment of a Sphere flat on one ſide convex on the other, muſt neceſſarily give the Object much clearer than the double Convex ones: For the Rays which penetrate the firſt Convexity, converge at the Center of the Segment, but the other Segment muſt ſcatter them, ac-

cording to all Rules of Opticks, and conſequently the Objects not appear diſtinct, neither can an elliptical or parabolical Figure be ſo true as the Segment of a Sphere, becauſe they have not one common Center to converge in.

CHAP. IX.

Squaring the ELLIPSIS.

FROM the Circle likewiſe may the Ellipſis be ſquared together with its other uſeful Properties diſcovered.

1. An Ellipſis is an imperfect Circle form'd from an Infinity of Centers, which may eaſily be ſquared without imagining it to be cut off from a Cone: For Inſtance, inſcribe a Square in a Circle (*Fig.* 1. *Pl.* 4.) as $abfd$ by drawing the two Diameters af and bd at Right-Angles, and joining their Ends with the Right-Lines ab, bf, fd, da, by 4 *Euc.* 6. acb being a Right-Angle, the Square of ab is equal to the Squares of ac and cb, by 47 *Euc:* 1. then ſuppoſe the Sides of this Square to be mov'd into a Rhombus, by imagining one Diameter

longer than the other, with the ſame Circle circumſcrib'd, it will form an oblong Figure, or an Ellipſis *a b d f*, (*Fig.* 2.) and *b d* in the Rhombus is equal to *a b* in the Circle. Hence the Angle *b c d*, in the Rhombus being a Right-Angle, the Square of *b c* and *c d* in the Rhombus are equal to the Squares of *a c* and *c b* in the Circle, being both equal to the Square of *a b*; and thus it is, let the Rhombus be never ſo much contracted, becauſe the Angle *a c b* is always right.

2. Hence the Squares of the Tranſverſe and Conjugate Diameters of the Ellipſis, are equal to the Square of the Diameter of the Circle, becauſe they are both equal to four times the Square of *a b*, conſequently the Square of the Diameter of the Circle, is a mean Proportional between the Squares of the two Diameters of the Ellipſis, and the Circumference of the Ellipſis is equal to the Circumference of the Circle, as the Circumference of the Rhombus is equal to that of the Square; for tho' the Surfaces decreaſe, the circumſcribing Lines do not: For inſtance, the Area of the Rhombus is leſs than the Area of the Square, as *b l* the perpendicular Height of the Rhombus is leſs than *b f* the perpendicular Height of the Square; for

b l is

bl is leſs than *bd*, which is equal to *bf* in the Circle, by 19. *Euc.* 1. Hence, tho' their Circumferences are equal, the Area of the Rhombus is leſs than the Area of the Square; for all Parallellograms that have equal Baſes, are to one another as their Heights, by *Schol.* 1. *Euc.* 6. Again, the Triangle *bmd* in the Ellipſis, is leſs than the Triangle *amb* in the Circle, for their Baſes are equal, but the perpendicular Height in the Circle, is bigger than in the Ellipſis; for *mn* in the Circle is Perpendicular to *ab*, but oblique in the Ellipſis; and thus the Area of the inſcrib'd Rhombus and Triangles decreaſe, as the Conjugate Diameter is ſhortned, tho' the circumſcribing Lines are of the ſame Length.

3. Hence 'tis eaſy to find the Circumference of an Ellipſis by the two Diameters; for multiplying one into the other, and extracting the ſquare Root, it gives the Diameter of a Circle whoſe Circumference is equal to that of the Ellipſis: For inſtance, let *bf* the conjugate Diameter be 10, and *da* the Tranſverſe 20, which multiplied are = 200, the ſquare Root is 14,14 which is the Diameter of the Circle, whoſe Circumference is equal to that of the Ellipſis,

and

and both of them eaſily obtained from the Principles of ſquaring the Circle.

4. To find the Focus of an Ellipſis. As the long and ſhort Radii or half Diameters are to one another as the Squares, ſubtract the Square of the leſſer from the Square of the greater, and extract the Square Root, it gives the Diſtance from the Center of the Ellipſis to the Focus on the greateſt Diameter: Let the Diameters be 10 and 20, which, ſquared, are equal to 25 and 100, ſubtract 25 from 100, remains 75, the ſquare Root is 8,66, which is the Diſtance from the Center to the Focus.

5. To draw a Tangent to the Ellipſis. Find the Focus's *e* and *d*, and draw the Lines *df* and *ef*, make *fa* equal to *fd*; and from the Nature of an Ellipſis *df* and *fe* are always equal, being equal to the longeſt Diameter; draw *da*, which divide in *b*, make *bc* perpendicular to it, and it will touch the Ellipſis in *f*, for *b* is without it, *da* is leſs than *ea*, by 19 *Euc.* 1. wherefore *b* is without the Ellipſis.

6. To find the Area of the Ellipſis, having the two Diameters given. This is done by the Rhombus inſcrib'd in the Ellipſis, and the Square in a Circle of equal Baſes; to find the Baſe of the

Rhombus Geometrically, by the two Diameters, add the Square of half the longeſt Diameter to the Square of half the ſhorteſt, and it gives the Square of the Baſe, extract the Square Root, and the Baſe is given; and the perpendicular Height is Geometrically found, as the Baſe is to half the longeſt Diameter, ſo is the ſhorteſt Diameter to the perpendicular Height; for the Triangle fbl is ſimilar to acf, the Angles at l and c being right, and the Angle at f common to both: Hence $fa : ac :: fb : lb$, let therefore the Baſe of the Rhombus, inſcrib'd in an Ellipſis, be 10, and its perpendicular Height 5, imagine a Square inſcrib'd in a Circle, whoſe Baſe is 10; the Height is the ſame; the Area of the Rhombus is 50; and of the Square 100; and if the Baſe of the Rhombus and Square are equal, the Circumference of the Circle and Ellipſis, in which they are inſcrib'd, are equal, by *Sect.* 1, and the Area's in Proportion to the Square and Rhombus, by *Sect.* 2.

7. Hence, to find the Diameter of the Circle from the Square, whoſe two Sides are 10 and 10, their Squares = 200 = to the Square of the Diameter of the Circle, the ſquare Root is 14,14, the Circumference of the Circle is

near 44, half the Radius 3 ½ or 3, 5 multiplied by 44 gives 154 for the Area of the Circle; wherefore by Proportion 100:154::50:77. the Proportion of the Area of the Ellipſis to the Area of the Circle, or the Ellipſis is equal to ½ of the Circle, as the Rhombus is one half of the Square; and whatever Proportion the Rhombus has to the Square, the ſame has the Ellipſis to the Circle. So much for the Ellipſis.

CHAP. X.

Reflections from the Circle *and* Ellipſis.

BUT now to proceed and make ſome proper Uſes of the Circle and Ellipſis together; and as King *Solomon* ſays, *A falſe Balance and a deceitful Meaſure are an Abomination to the Lord.*

1. It will be neceſſary to have recourſe again to the Chapters of Mechanicks, and the Ellipſis, in order to demonſtrate how eaſily the World may be impos'd upon, with reſpect to Weight and Meaſure. Let therefore *a b* (*Fig.* 5. *Pl.* 3.) be the Balance, and ſuppoſe the Weight

were put in *a*, and the Commodity in *d*, 'tis plain the Commodity would be $\frac{1}{2}$ leſs than its juſt Weight; for *c d* has but half the Velocity or Length as *c a* has: And thus the Weight will be more or leſs in Proportion to their different Lengths.

2. The ſame is demonſtrated about Meaſure from the Ellipſis; for of all perimetrical Figures, whoſe Circumferences are equal, the Circle includes the biggeſt Surface; and conſequently a cylindrical Veſſel will contain more than an Elliptical one of an equal Girt and Height.

3. Take therefore the Circle and Ellipſis of an equal Circumference, as mentioned before, the Area of the Circle is 154, and of the Ellipſis 77; let the Height be 8, which multiplied into the Areas, the cubical Contents of the cylindrical Veſſel is 1232, and of the Elliptical one 616, juſt $\frac{1}{2}$; and thus the Veſſels will be in the ſame Proportion with the Rhombus and Square.

4. From hence we may learn, that if a Ship be made too long for the Breadth, it leſſens the Burthen, and that to make it uſeful for Burthen and Sailing, it muſt be made narrow at the Ends and broad in the Middle.

CHAP.

CHAP. XI.

Further Uſefulneſs of the Ellipſis.

THE extenſive Uſefulneſs of the Ellipſis may likewiſe be ſhew'd in other Things.

1. As a Reaſon may be given why a Body that moves in an Ellipſis by even Motion, or by cutting off equal Portions, ſhall ſeem to move faſter and ſlower to a Spectator in the Center.

2. Suppoſe an Ellipſis, as *age* (*Fig.* 4. *Pl.* 4.) and a Circle made upon the Center *c*, let *ge* be equal to *if*, a Body that moves in the Ellipſis from *g* to *e*, will ſeem to move faſter than one in the Circle, from *i* to *f*, becauſe as they are ſuppos'd to have the ſame Velocity, and to cut off equal Arches, the Body in the Ellipſis will appear to be at *h*, when the other is at *f*; hence its apparent Velocity is greater, and which is greateſt at the ſhorteſt Diameter: For ſuppoſe *ad* and *ge* in the Ellipſis, and *ab*, and *if* in the Circle to be equal, *ge* is greater, with reſpect to *if*, than *ad* is of *ab*; for *ad* being equal to *ge*, but the Sides *ac* and *dc* are longer than

cg and *ce* from the Nature of an Ellipſis; conſequently, the Angle *gce* is bigger than *dca*, by 21 *Euc.* 1. and hence cuts off a greater Arch.

3. Hence likewiſe the greateſt Force of a Body that moves in an Ellipſis, or Parabola, by equal Moments, is at the ſhorteſt Diameter, and the Force is in Proportion to the Difference of the Diameters; and Bodies aſcending and deſcending, or that move parallel to the Horizon in a ſtrait Line, accelerate and retard their Velocity, in Proportion to the verſed Sines of their Motion, or Squares of the Diameter of Diſtance; becauſe they move in the Diameter of a Circle, which is a double verſed Sine to the Circumference: For Inſtance, Suppoſe a Body to move by equal Moments of Time, in the Diameter *cd* (*Fig.* 10. *Pl.* 2.) from the Circumference to the Center of the Circle *abd*, divide the Quadrant *dba* into 90 Degrees, equal to the Moments; let the Increment of the Angle *bce*, be equal to the Moments or Degrees, *be* being the Sine, *de* is the verſed Sine; and thus as the Angle increaſes as the Degrees or Moments are, the Motion will increaſe as the verſed Sines, and in moving to the Center, is equal to 90 Degrees of verſed

Sines, or the Motion would not be equal to the Increment of the Angle, contrary to the Suppoſition: Hence then a Ball fired out of a Cannon in a ſtrait Line, the greateſt Force is at the Center of the whole Diſtance, that the Ball ranges, and before it comes to the Center it accelerates the Motion, and from the Center retards it as the verſed Sine; but if it be in an Ellipſis or Parabola, the Angle of Elevation, by which the Ellipſis, *&c.* is known, will determine the greater or leſſer Force that the Ball will have at the ſhorteſt Diameter, the greateſt Force being in the Diameter or ſtrait Line.

4. From hence it may be demonſtrated, that no progreſſive Motion or Body can be propagated infinitely; for all Motion is either Circular, Elliptical, or in the Diameter of a Circle or ſtrait Line; if it is propagated circularly, it moves round a Center; if elliptically, it increaſes and decreaſes its Velocity in Proportion to the Nature of the Ellipſis; and if it is propagated in the Diameter of a Circle or ſtrait Line, it increaſes the Velocity as the verſed Sines of the Diameter, till it comes to the Center, and then decreaſes again; wherefore if a Body is propagated infinitely in the

Diameter of a Circle or ſtrait Line, it muſt either be done by an infinite Force, or the Circle muſt be infinite; if it be done by an infinite Force, the Velocity will be inſtantaneous; for the Velocity is in Proportion to the Force, by *Sect.* 2. *Chap.* 5. But this is a Principle wholly inconſiſtent with Matter and progreſſive Motion; neither can it be done in the Diameter of an infinite Circle, for an infinite Circle cannot be aſſigned, ſince it may be enlarg'd to Eternity, which can never be; for as Eternity paſt had no Beginning, ſo Eternity to come will have no End, hence will never be; or ſuppoſe it propagated infinitely in the Diameter of an infinite Circle, it muſt paſs through the Center before it comes to the Circumference on the other Side; and this Part of the Motion would be performed in Time, and the other Part of the Diameter or Radius being equal to the Firſt, muſt likewiſe be done in Time; but 'tis poſſible to add more Time, and the Radius lengthned, and this may be done to Eternity; hence here would be a Radius and Part of a Circumference one way, and an infinite one the other, which is an Impoſſibility in the ſame Circle, and the accelerating and retarding Motion of all Bodies in a ſtrait Line, plainly demonſtrates that it has an End.

5. Hence, whatever is infinite muſt be eternal and immaterial, for no Matter can be infinite, becauſe it has Figure, and hence a Center and Circumference and is terminated; and then it muſt be eternal and immutable, and without Succeſſion and ſupported by a different Subſtance than what Matter is, for finite and changeable Accidents are ſupported by material and diviſible Subſtance; but an infinite and immaterial Being by immaterial and indiviſible Subſtance; for though the Subſtance of Matter is diviſible, yet the Subſtance of Spirit is indiviſible, as Thought or Conſciouſneſs cannot be divided, hence neither the Subſtance that ſupports it; then the Properties of material Subſtance and Accidents are different; for Subſtance cannot receive more or leſs, or be more or leſs Subſtance as ſuch: Hence if a material Body were divided into ten thouſand Parts, and each of thoſe Parts were carried into ſo many different Places, the whole Subſtance of the Body would be in each of thoſe ten thouſand Places at the ſame Time, for material Subſtance is the Whole in the Whole, and the Whole in every Part, as the whole

Subſtance of Bread is in every Part; and the ſame may we ſay of the Subſtance or Genius of every material Being, for what is extended or viſible is not the Subſtance but Accidents.

6. Hence likewiſe Space or Capacity is not infinite, as being terminated by Figure, neither is Time eternal, as being meaſurable by Number, tho' one Part of Space or Time cannot be ſeparated from another, as Matter and Number are ſeparable, yet the Parts are really diſtinct from one another, whither it be extrinſick or intrinſick Duration or Space; for Motion is the Meaſure of extrinſick Duration, and if there had been no Motion from the Creation to this Time, the intrinſick Duration would have been equally long and diviſible into Periods, as it is now it is divided; ſo intrinſick Space or Extenſion may be conceived without Body and diviſible into Parts; for all Geometrical Lines or Surfaces, either ſtrait, circular or ſpherical, are pure Extenſion and diviſible into Parts; but theſe Periods or Parts are wholly inconſiſtent with Eternity and Infinity, otherwiſe Eternity paſt Yeſterday is leſs than Eternity paſt to Day; and conſequently Eternity to come Yeſterday, was more than Eternity to come

to Day; take the Mean between theſe two Days, the two Eternities are equal; and it is the ſame if any other two Days are taken, either Millions of Years paſt, or Millions of Years to come, which would make the ſame Eternity to be more and leſs equal and unequal, and all the poſſible Ways of Contradiction, and the ſame Contradictions are in infinity or infinite Extenſion: For ſuppoſe the Line *ab* (*Fig.* 4. *Pl.* 1.) infinitely extended both ways, let *e* be the Middle, divide it again in *c* and *g*, conſequently *g b* is equal to *g a*, and *a d* equal to *a c* the Parts and Wholes, becauſe they are all infinite by Suppoſition; and hence *g*, *c*, *e* are all Middles of the ſame Line, which is impoſſible. Theſe Contradictions do not happen in ſtated Periods and finite Extenſion, for every the leaſt Addition or Diminution makes them longer or ſhorter, conſequently Eternity and Infinity do not conſiſt of ſucceeding Periods and extended Parts; if they did, the Wholes muſt have the ſame Properties with reſpect to Multiplication and Diviſion that the Parts have, as the Whole is equal to all the Parts, by 19 *Axiom*, *Euc.* 1.

7. The true Notion therefore of Eternity is, that it comprehends every

Thing paſt, preſent and to come, as well as Infinity includes all Extenſion, and neither can receive Addition or Diminution; and hence then, as neither Matter, Space, or Time, can be eternal or infinite, and ſomething muſt neceſſarily be ſo, which hath neither Parts or Succeſſion, and that hath created all theſe Things, for they could not be by Chance, ſince that at the moſt can only alter the Modus of pre-exiſtent Beings, the Motion of the World could not happen by Chance, ſince Motion is not eſſential to Matter; for its Tendency would be equally every way round the Body, and a Body that has an equal Tendency to move every way, will never move without a firſt Mover to determine its Motion; hence the World could not be form'd by a fortuitous Concourſe of Atoms, neither could theſe Atoms be eternal, becauſe they muſt exiſt in Space and Time; the animal Motion is determin'd by the Object, tho' we are conſcious that the Will is free to follow the Determination of the Judgment; but the Motions of the Will are quite different from the Motions of Matter, which is only progreſſive, and conſequently the Soul cannot be Material; and in thus altering the natural State of Matter does the Free-Agent in ſome Meaſure partake of the Divinity.

8. And then this ſupreme Something and Creator muſt be Eternal, Infinite, Intelligent, Independent, and neceſſarily one.

Eternal, becauſe ſomething muſt neceſſarily be Eternal, and neither Time, Space, or Matter are ſo.

Infinite, becauſe he comprehends every Thing that he has created, and nothing can be where he is not; for if it was, it muſt be independent of him, and conſequently not created, which is impoſſible; hence the Creator is not Space, becauſe Space is not infinite.

Intelligent, becauſe he has created intelligent Beings; we are conſcious of our own Exiſtence, but we know not how we came into Being, we did not form our ſelves, and conſequently our Exiſtence muſt either depend upon immediate Creation or a Succeſſion of Cauſes, till we come to the firſt Cauſe, ſince an eternal Succeſſion is a Contradiction.

Independent, becauſe every thing elſe depends upon him; for it requires the ſame Power to ſupport the World in Being, as it did at firſt to create it out of nothing, otherwiſe it would exiſt independent of the Creator, and conſequently eternal, which is impoſſible; we cannot ſee him any more than we can perceive the Operations of the Soul by our eternal Senſes, and as he created the Soul, he muſt be infinitely more pure than that is.

And neceſſarily one; for as created Spirits are indiviſible, as the Accidents, Conſciouſneſs, *&c.* are indiviſible, and hence the Subſtance; ſo much more muſt the Subſtance of the ſupreme Spirit and Creator of all be indiviſible, as being infinite, and conſequently without Extenſion. Beſides, if there were two ſuch Beings of all Perfections, the one would have all the Perfections that the other has, beſides its own Perfections, and the other would have all the Perfections that the firſt had, beſides its own Perfections; and, conſequently, there would be a mutual Superiority which is a Contradiction, for there cannot be two Wholes of all Perfections. And tho' this Divine Being is one in Eſſence, yet he is three in Perſons, which is believ'd upon the Evidence of Revelation, than which no moral Evidence can be ſtronger. For as we believe that *Cæſar*'s *Commentaries* were written by *Julius Cæſar*, and conſequently that the Facts were moſtly true; ſo we have much more Reaſon to believe that the Books of *Moſes* were written by him, becauſe here was a

Succeſſion of Men for near two thouſand Years, who all atteſted the Truth of it by Miracles; beſides, a whole Nation of People continue to this Day, whoſe Government is founded upon his Principles, and preſerve a continual Remembrance of him by their Yearly Feſtivals. And the ſame Evidence have we, with reſpect to the Truth of the Chriſtian Religion: We have the Facts of our Lord and Saviour Jeſus Chriſt recorded by four different Authors, beſides the preceding Propheſies, which were all fulfilled in him; and the living Evidence of ſome of all Nations who Yearly celebrate their Feſtivals in Honour of him; and if the Facts are true, the Miracles and Propheſies muſt neceſſarily be ſo; and theſe cannot be done by an ordinary Power, but it muſt immediately be deriv'd from the ſupernatural Power of God. Theſe Miracles our Bleſſed Saviour did by his innate Power, becauſe he ſeldom invok'd any other, and which have been done ſince in his Name, as mention'd in the Acts of the Apoſtles: Hence, ſo many Books and Authors, tending to the ſame End, of promoting the Good and Happineſs of Mankind, and all working Miracles for the Truth of their Relations, could not be in an Error.

ror. The *Mahometan* Religion was fully compleated by *Mahomet*, which was founded by the Sword, without Miracles, and never confirmed by any ſince. The ſame we may ſay of all other Religions, which tho' their Facts are true, yet for want of Miracles they are no other but Impoſtures to us. Having therefore this ſtrong moral Evidence of the Truth of the Revelation, we believe the incomprehenſible Myſtery of the Bleſſed Trinity; for the *Word*, as mentioned in the firſt *Chap.* of *Geneſis*, which ſpoke the World into Being, and the Spirit of God which mov'd upon the Face of the Water, are plainly diſtinguiſh'd; and St. *John*, in the firſt *Chap.* of his Goſpel, ſays, *In the Beginning was the Word, and the Word was with God, and the Word was God. The ſame was in the Beginning with God. All things were made by him; and without him was not any thing made that was made:* Again, *And the Word was made Fleſh, and dwelt amongſt us.* Here he plainly diſtinguiſhes God and the Word, and that the Word was God, and made Fleſh, and that all Things were made by him that were made, conſequently he could not be made himſelf; and, ſuppoſe, inſtead of the Word was God, it ſhould be the Word of God, the ſame Power of Creation, as diſtinguiſh'd from God, is aſcrib'd to him; conſequently, he was not created but eternal, and hence God. Baptiſm, by which Sin is pardon'd, is done in the Name of the Father, and of the Son, and of the Holy Ghoſt; and in the Apoſtles Creed we profeſs the ſame Act of Faith, in the Father, Son and Holy Ghoſt, the three Perſons of the moſt Bleſſed Trinity, which would be Idolatry, if the Son and Holy Ghoſt were not God. But how three Divine Perſons ſhould be one God, we can no more comprehend, than we can comprehend how an unextended indiviſible Being ſhould create and contain in his Preſence all Nature that is extended and diviſible.

9. Hence likewiſe, it may be argued, that as Matter and Space are diſtinct Exiſtences, becauſe Matter is moveable but Space is not, 'tis highly probable that the Syſtems of the World revolve in pure Space, or only Rays of Light in Tranſitu, or their Motions could not anſwer to any ſecondary Laws of Motion, ſo as to preſerve the ſame conſtant regular Revolutions and Periods; for their Motions cannot be perform'd by any mechanical Principles, ſuch as fluid or ſolid Spheres; the Doctrine of Comets

wholly deſtroys them, for theſe go beyond the Sun, and penetrate the Spheres with their contrary Motion, beſides the contrary Motions that muſt neceſſarily happen in theſe inferior Spheres; for as the Moon goes round the Earth, and the Earth round the Sun in one Part of the Moon's Circle, the Motion of the Sphere is directly contrary to that of the Earth's, and conſequently muſt leſſen or wholly deſtroy their Motion; the ſame happens in the Satellites of *Jupiter* and *Saturn.*

10. From the Circle and Ellipſis likewiſe is demonſtrated the Nature of Gravity, as that all Bodies which move in ſtrait Lines, accelerate their Motions upon the Diameters of their Circles of Force or Diſtance, in Proportion to the verſed Sines, till they come to the Center, and then decreaſe again; but tho' each of theſe Parts gravitate thus, yet take Gravity in general, it is in a triplicate Ratio, or as the Cubes of the Diameters of the Spheres, becauſe the Gravity of a Globe is ſpherical; hence the Gravity of a Body at one Foot Diſtance from the Center, is eight Times as much as one at two Foot Diſtance, or 8 : 1. and the other intermediate Diſtances, decreaſe as the Diviſions on a Monachord, being the remaining Surds

after the Cube Root is extracted; and thus ſuppoſing it to be a Sphere or Syſtem of Bodies, as that of the Sun and Planets, if they are acted by a Principle of Gravity, it will be as the Cubes of the Diſtance from the Center, or the Surds of Cubes.

11. But then as the Action of Gravity is wholly independent of Matter, otherwiſe its Tendency would be equally all round every Point of Matter, and conſequently muſt deſtroy itſelf; neither can we obſerve one Body to act upon another at a Diſtance, unleſs it is by means of intermediate Bodies, which is different from the Actions of Gravity; ſo why ſhould we multiply Entities without Neceſſity, ſince the Frame of Nature may better be accounted for by two Principles, *viz.* Matter and ſimple Motion, than by the other Principles of the Imagination? For Inſtance, ſuppoſe the Elements to be created, as mentioned in the firſt Chapter of *Geneſis*, and in the Chaos a Center appointed for every Body or Sphere that is now in the World, and the Matter that compos'd that Body to move all round in a ſtrait Line to the Center, the Velocity of that Motion would increaſe as the verſed Sines, and the Matter which meets in the Cen-

ter would have the greateſt Velocity, which Velocity decreaſes in Proportion to the Diſtance from the Center; and this muſt neceſſarily keep the Globe or Sphere together, unleſs torn aſunder by a greater Force: For all Matter retains the ſame Force or Tendency as was firſt impreſs'd upon it, as nothing can deſtroy it any more than it can the Body, but the Power that firſt gave it, and which upon the Removal of any Obſtacle returns again to its firſt Nature. Let therefore *e l g* (*Fig.* 5. *Pl.* 4.) be the Globe or Sphere, and the Matter of that Sphere before it was put into Motion, or the Spirit of God mov'd upon the Face of the Waters be *a b i d*; and ſuppoſe it to move all round in a Diameter or ſtrait Line to the Center *c*, the Velocity increaſing as the verſed Sines, the ſame Force continuing when two Bodies meet in the Center; and as their Velocity is greateſt in the Center, the Coheſion will be ſtrongeſt, and the Force and Coheſion of the other Matter will decreaſe as the verſed Sines, and in order to make the Body more compact, 'tis but imagining the outer Sphere to be larger, and conſequently it muſt be a greater Force to drive the Matter at a greater Diſtance, and the Velocity will be in Proportion to the verſed Sines, which are in Proportion to the Diameter.

12. Hence, the bigger the Body is, the greater is the Tendency to the Center, and that a greater or leſſer Gravity is not founded upon the Quantity of Matter, but on the Bigneſs of the Sphere, from whence the Force was firſt impreſs'd upon Matter; and hence 'tis probable that the whole Creation is the Sphere or Circumference of Fire; and this is the Cauſe of its Motion being ſo quick when it meets no Hindrance.

13. Suppoſe, again, when the Syſtems of the World were erected, that each Body had a double Force impreſs'd upon it, one towards the Center, about which they revolve the other Side-way at right Angles, or in a Tangent Line; this would put the Body in a circular Motion like a Stone in a Sling, whirl'd round by the Hand; for Inſtance, let *t t* (*Fig*. 6. *Pl*. 4.) repreſent the Circle, and *a t*, *a t*, be two Tangent Lines, let *c t*, *c t* two Radii meet them at right Angles, if the Force *a t* be equal to *t c*, the Angles will always be right, and conſequently muſt neceſſarily from a Circle; and if the Forces are in Proportion to the Diſtance from the Center or Radius, the Squares of the

Circles will be as the Cubes of the Diſtance; for the Diviſions on the Monachord decreaſe by equal numerical Diſtances, as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ or $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, *&c.* theſe are the remaining Surds of Cubes, and the Circles are to one another as Squares. And from theſe Principles 'tis eaſy to account for the Motion of a Body in an Ellipſis; for 'tis only imagining the two Impreſſions to be made in an Angle, bigger than a right one, and in Proportion to the Nature of an Ellipſis: For Inſtance, (*Fig.* 7. *Pl.* 4.) let *tt* be the Ellipſis *at*, *at* two Tangents, *tc*, *tc* two Radii; and ſuppoſe the Angle *atc*, *atc* to be always in Proportion to the Nature of the Ellipſis, 'tis plain if the Force *at* and *tc* be equal, it will always preſerve that Angle, and conſequently form an Ellipſis; but from the Principles of Gravity and projectile Motion it cannot be accounted for, becauſe there are ſeveral Centers in an Ellipſis, which will every Moment require a different Degree of Gravity and projectile Motion; neither can the Motion of a Body be accounted for from theſe Principles, that does not move round the Center of its Circle, for the greater and leſſer Diſtances would every Moment require a different projectile Motion to anſwer to the ſeveral Degrees of Gravity. For Inſtance, (*Fig.* 1. *Pl.* 3.) ſuppoſe *fz* the greateſt Diſtance, and *fk* the leaſt, let the Difference be 9:10; and ſuppoſe Gravity decreaſes as Squares, the Proportion is 81:100, which muſt neceſſarily require a different projectile Motion every Moment; but then as Gravity decreaſes as Cubes, the Proportion is 729:1000, and in Ellipſis's and Parabolas it is infinitely more. Theſe Difficulties do not occur in the two Impreſſions.

14. As for the Motion of the Tides which are obſerved principally to be govern'd by the Motion of the Moon, theſe ſeem rather to depend upon the near Communication of the Moon acting like a Load-ſtone, than upon any Principles of perpendicular Preſſure or Attraction; for as the Action of the Load-ſtone is probably owing to the Æther, which ſuppoſe to be a Body or Fluid ſo rare, that another Body may paſs through it with little or no Reſiſtance, as 'tis probable Sounds are propagated in Æther and not in Air, becauſe contrary Winds do not retard the Propagation, and all the other Senſes may be more or leſs affected by it; let this Æther be at reſt, or as the Earth with the Atmoſphere is whirl'd round the

Axi

Axis from *Weſt* to *Eaſt*, the Motion of the Æther is from *Eaſt* to *Weſt*; and ſuppoſe the Load-ſtone to have the Power of retaining this Æther and communicating it to the Needle, and as that hangs in *Equilibrio*, the Reſiſtance of the Air is equal, which with the Æther, makes it point *North* and *South*, like a Globe put in ſtanding Water, and turn'd round the Axis, a Needle fix'd in *Equilibrio* will always point to the Poles; and as for the attractive Quality of the Load-ſtone, it may be likened to two Irons heated by the Fire, which eaſily join together by means of the Fire as the other doth by Æther, and that Point of the Load-ſtone which is neareſt the Steel always attracts the ſtrongeſt. Now, ſuppoſe the Earth and Moon to retain this Æther, 'tis plain that as the Moon is neareſt to that Part of the Earth to which it is perpendicular, by 8. *Euc*. 3. the Æther muſt act moſt ſtrongly; and this rarifying the Atmoſphere, which being a Fluid, will preſs on every Side towards the Rarefaction, and neceſſarily raiſe the Water, as it is raiſed by Rarefaction in Pumps. For Inſtance, let the Moon be *o* (*Fig*. 8. *Pl*. 4.) the Atmoſphere *f a b*, let the Earth be *l h*, where the Moon is perpendicular, by means of its Æther mixing with that of the Earth it rarifies the Atmoſphere, which preſſing on every Side muſt raiſe the Water, and that continually diſperſing according to the Motion of the Moon, will affect the whole Sea; and as there is no Preſſure on the oppoſite Part of the Earth, hence the Preſſure of the Sides muſt neceſſarily affect that Part which is the Cauſe of two Tides in one Revolution of the Moon; the irregular Tides are probably owing to the different Turnings of the Coaſts, that the Water paſſes by together with the meeting of oppoſite Currents, becauſe the Water muſt be in a continual Flux and Re-flux; and this different Flux of the Atmoſphere is the Cauſe of the Winds, which are generally regular where the Moon is perpendicular.

15. From the Circle and Ellipſis likewiſe we may argue, that a Body will not always keep the ſame uniform Motion, unleſs it be in a Circle, but accelerates it till it comes to a Center, which Center is in Diſtance from the Circumference in Proportion to the Force that is firſt impreſs'd upon it, and from the Center retards the Motion, and that the natural Motion of a Body is in a Circle; theſe are the Ways of Motion, which,

without Reſiſtance and Allowances, muſt be made according to the Medium it moves thro', and the Reſiſtance it meets with.

16. Hence likewiſe may the Nature of the Ellipſis that a Body revolves in be known by two or three Obſervations of the Longitude, ſuppoſing the Obſerver to be in the Center of the Ellipſis, from two Obſervations, if the whole Revolution be known; and from three, if not known at equal Diſtances of Time: For Inſtance, let the Revolution known be that of the Moon, which according to the Tables, is 27 Days, 7 Hours, 43 Minutes; and this is the Nature of an Ellipſis, that a Body moving in it will appear to a Spectator in the Center, to move faſter in one Quarter, and ſlower in the next; and faſter in the Third, and ſlower in the Fourth. Hence then, if the Moon moves in an Ellipſis, as its whole Revolution is 360 Degrees, one Quarter is 90, which is perform'd in about 6 Days 20 Hours, which is near 13° in 24 Hours; but in order to make Obſervation, theſe Calculations ought to be done to the greateſt Exactneſs: Then, ſuppoſe it found by Obſervation to move more or leſs than 13° in 24 Hours, take the Difference, which ſuppoſe to be 1 Degree, conſequently it muſt gain or

loſe near 7 Degrees in that Quarter, which it gains or loſes again in the next; to find the Nature of the Ellipſis, the Degrees which are 90 in the Ellipſis, or *age* (*Fig.* 4. *Pl.* 4.) are 97 on the Circle, or *aif* which makes the whole Circle to be 388, and the Proportion to the Diameter is 9 : 28; take therefore ½ the Diameter or 61, for 28 : 9 : : 388 : 122 and ½ is 61; then by Proportion 97 : 61 : : 90 : 56 which is near the Difference between the long and ſhort Diameter, or the longeſt Radius is 61, and the ſhorteſt 56; ſo likewiſe by three Obſervations the Increment or Decrement of any Motion is known whoſe whole Revolution is not given, becauſe in three Obſervations the Difference of the Velocity or Slowneſs is known; and hence 'tis eaſy to determine the Length of the whole Revolution. So much for the Circle and Ellipſis.

CHAP.

CHAP. XII.

To double the CUBE.

TO double the Cube Geometrically, or find two or more mean proportional Lines between two Lines given.

In *Cor.* 33. *Euc.* 11. it is, if there be four right Lines in continual Proportion, as the firſt is to the fourth, ſo is a Parallelopipedon on the firſt to a Parallelopipedon on the ſecond; hence if the fourth Line be double to the firſt, a Parallelopipedon upon the ſecond is double to one of the firſt, which is doubling the Cube.

To do it Geometrically, draw a Line as *cg* (*Fig.* 6. *Pl.* 1.) dividing it into equal Parts, by *Sch.* 10. *Euc.* 6. at the fourth Diviſion, let *c a* be perpendicular to *c g*, and double and parallel to *g k* draw the Line *a k b*; let *cg* meet it at *b*, and *d e*, *f l* being drawn Parallel to the other two Lines are two mean Proportionals between *gk* and *c a*; for *b c a*, *b f l*, *b e d*, *b g k* are ſimilar Triangles, having their Angles equal; hence *b c* : *c a* :: *b f* :: *f l* : *b e* : *e d* : : *b g* : *g k*, and the Differences between *b c*, *b f*, *b e*, *b g*, are equal by Conſtruction, conſequently the

Differences between *c a*, *f l*, *e d*, *g k* are equal, and *c a* : *f l* :: *e d* : *g k*, but *c a* is double to *g k* by Conſtruction; hence a Solid made upon *de*, is double to one upon *g k*, and if *c a* had been triple, quadruple, &c. to *g k*, one made upon *de* would be triple, quadruple, &c. to one made upon *g k*, and then not only two mean Proportionals may be found, but alſo any Number, by dividing the Line *g c* into ſo many equal Parts. So much for the firſt Part.

A MISCELLANY OF MATHEMATICKS.

PART II.

CHAP. I.

Of TRIGONOMETRY.

AN eaſy Method to calculate Triangles by the Proportion of the Sides to the Angles.

1. The Myſtery of practical Trigonometry, plain and ſpherical, depends upon the fourth Propoſition of the ſixth Book of *Euclid*, That if two Triangles have their Angles equal, the Sides are in Proportion; and if the Sides are

in Proportion the Angles are equal: Hence the Triangle taken, and the Triangle calculated, are ſimilar.

2. The three noted Caſes of the Equality and Proportionality of Triangles are.

If two Sides and one Angle between them, in one Triangle, be equal, or proportional to two Sides, and one Angle between them in another, the Triangles are equal or ſimilar, by 4, 1. and 4. *Euc.* 6.

If two Angles and one Side between them in one Triangle, be equal or proportional to two Angles, and one Side between them in another Triangle, the Triangles are equal or ſimilar, by 26, 1. and 4. *Euc.* 6.

If all the three Sides of one Triangle be equal or proportional to all the three Sides of another, the two Triangles are equal or ſimilar, by 8. *Euc.* 1.

3. In theſe three Caſes are contained all the Ways of calculating Triangles; for in every Triangle there are ſix Parts, three Sides, and three Angles, and if there be any of theſe three given, the reſt may be found, unleſs it be when the three Angles only are given. But the Tediouſneſs of the Operations, according to the common Method, beſides the Inconveniencies of always having

Tables or Scales, was the Cauſe of finding out this Calculation of Triangles, by the Proportion of the Sides to the Angles.

4. There are three Sorts of Triangles, with reſpect to their Angles, right, acute and obtuſe, which have all a different Proportion of their Sides to the Angles.

In right angled Triangles the Square of the Hypothenuſe is equal to the Squares of the other two Sides, by 47. *Euc.* 1. and by 32. *Euc.* 1. the three Angles of a Triangle, are equal to two Right: Hence, the two acute Angles in a Rectangle Triangle are equal to the right Angle, as the Squares of the Sides are equal to the Square of the Hypothenuſe, and the Squares of the Sides are to one another as the Sides are, by 24. *Euc.* 1. and *Theor.* 48. *Euc.* 1.

In obtuſe angled Triangles the Square of the biggeſt Side is equal to the Squares of the other two Sides, and twice the Rectangle under the Baſe, and a Line between that and a Perpendicular from the oppoſite Angle, by 12. *Euc.* 2. Hence, the Squares of the Sides which include the obtuſe Angle, are in the ſame Proportion to the Angles oppoſed to them, as the Square of the greater Side, together with the two Rectangles, is to the obtuſe Angle, or the Squares of one Side is to the oppoſite Angle, as the Square of the longeſt Side with the two Rectangles is to the obtuſe Angle.

In acute angled Triangles the Square of the Side ſubtending the acute Angle is leſs than the Sum of the Squares of the other two Sides, by double the Rectangle under the whole Baſe, and a Segment of the Baſe form'd by a Perpendicular, which is next the acute Angle, by 13. *Euc.* 2. Hence, the Square of one Side together with one Rectangle is to the Angle oppos'd, as the Square of the Side oppoſing the acute Angle is to the acute Angle; for the two Rectangles under either Side are equal, for in the Triangle *a b c*, (*Pl.* 4. *Fig.* 9.) let the two Perpendiculars be *a e* and *b d*, the Triangle *c a e* is ſimilar to *c b d*, the Angles at *e* and *d* being right, and the Angle at *c* common; hence *c d* : *e b* :: *e c* : *d a*: that is, the Rectangle under *c b e*, is equal to the Rectangle under *c a d*.

But in obtuſe and acute angled Triangles, there is no Occaſion to take notice of the Rectangles, unleſs it be when the three Sides only are given. Hence, the Proportion is in the Squares of the Sides, and conſequently the Sides, by *Theo.* 48, 1. and 18. *Euc.* 1. This includes all Sorts of Triangles, and the Proportion of the Sides to the Angles. And thus, by the Squares of the Sides may any Caſe in Trigonometry be reſolv'd, which is much more expeditious and certainly truer than by Sines and Tangents; for theſe are found out by their Squares, beſides the many Errors that may be committed in making and printing ſuch large Tables.

CHAP. II.

Of the Caſes in plain TRIANGLES.

BUT now to proceed, and give the uſual Caſes in plain and ſpherical Triangles: In plain Rectangle Triangles there are ſeven Caſes.

CASE I.

Given one Side and two Angles, required the other Side.

Let the Side given be (*Fig.* 10. *Pl.* 4.) $cb = 20$, the Angle $a = 26$ $c = 64$, and the Side required ab.

Square the Side $cb = 400$, and by Proportion $26 : 400 :: 64 : 984 +$, the Square Root is $31{,}3 = ab$ the Side required.

CASE II.

Given the ſame Side and Angles, required the Hypothenuſe.

By Proportion 26 : 400 : : 90 : 1384. the Square Root is 37, 2 = *ac* the Hypothenuſe.

CASE III.

Given the Hypothenuſe and one Side, required the two Angles.

Let the Hypothenuſe given be *ac* = 37 +, and the Side *ab* = 31 +, and the two Angles required be *a* and *b*.

The Sides ſquared are 1384 and 984 by Proportion 1384 : 90 : : 984 : 64 = to the Angle *c*, and ſubtracted from 90, leaves the other Angle *a* = 26.

CASE IV.

Given the ſame Sides, required the other Side or *c b*.

Find an Angle by Caſe 3, then by Proportion 64 : 984 : : 26 : 400, the Square Root is 20 = *c b*.

CASE V.

Given the two Sides, required the two Angles.

Square the two Sides *ab* = 984, and

$cb = 400$, which added together $= 1384$ then by Proportion $1384 : 90 :: 984 : 64$: which ſubtracted from 90, leaves the other Angle $= 26$.

CASE VI.

Given the two Sides, required the Hypothenuſe.

Find an Angle by Caſe 5, which let be $c = 64$, then by Proportion $64 : 984 :: 90 : 1384$ the Square Root is $37 + =$ the Hypothenuſe *a c*.

CASE VII.

Given the Hypothenuſe and the two Angles, required a Side or *a b*.

By Proportion $90 : 1384 :: 64 : 984$, the Square Root is $31 +$.

All theſe ſeven Caſes may be applied to Navigation, Surveying, taking Heights or Diſtances, Fortificntion and Gunnery, with much more Truth and Expedition than they can be done with Logarithms, and Sines, and Tangents.

CHAP.

CHAP. III.

Of ſpherical TRIGONOMETRY.

IN oblique angled Triangles there are 6 Caſes, and under theſe are included all obtuſe and acute angled Triangles.

CASE I.

Given two Sides and one Angle, required the other Angle.

Let the two Sides given be (*Fig.* 11. *Pl.* 4.) $ba=20$ and $bc=37$, and the Angle $bca=14°, 30'$ ſquare the two Sides $ba=400$ and $cb=1369$, then by Proportion $400:14°,30'::1369:49°,30'$.

CASE II.

Given the ſame two Sides and the Angle $a=49°, 30'$, required the other Side ac.

Find the Angle $c=14°, 30'$ by the firſt Caſe, and conſequently the Angle $cba=116$ is given, by 32. *Euc.* 1.

Square the two Sides, which added together are $=1769$, then by Proportion $116°:1769::180:2745$ the Square Root is $52=ac$, the Proportion is taken from 180, becauſe the Square of the greater Side is equal to the Squares of the two other Sides and two Rectangles, equal to the Complement of two Rectangles: This Caſe cannot be ſolv'd by Sines and Tangents.

CASE III.

Given two Angles and one Side, required the other Side.

Let the two Angles given be *c*=14° 30′, and *a*=49°, 30′, and the Side *a b*; required *b c*. Square the Side, and by Proportion 14, 30:400::49° 30′: 1414+, the Square Root is 37+=*c b*.

CASE IV.

Given two Sides and the Angle between them, required the other two Angles.

Let the two Sides given be *a b*=20, and *b c*=37+; and the Angle *d c a*= 116, required the Angles *c* and *a*.

Square the two Sides, and by Proportion the Sum of the Squares of the two Sides is to the Sum of the two Angles, as the Square of one Side is to one of the Angles, or 1769:64°:: 400: 14°,30′.

CASE V.

Given the ſame two Sides, and the Angle between them, required the other Side.

Square the two Sides, adding them together, then by Caſe 2.

116 : 1769 :: 180 : 2745, the Square Root is 52.

CASE VI.

Given the three Sides, required the three Angles. Here the two Rectangles come in to be added in obtuſe, and ſubtracted in acute angled Triangles; and in order to find them ſquare the three Sides $acq = 2704$, $bcq = 1369$, and $baq = 400$; add the two leſſer Sides ab and $bc = 1769$, which ſubtracted from 2704, leave 935 for the two Rectangles; then add all the Squares and two Rectangles together, and by Proportion, as all the Squares and Rectangles are to all the Angles, ſo is the Square of one Side to the oppoſite Angle.

$$\begin{array}{r} ca = 2704 \\ ab + bc = 1769 \\ 2\,\text{Rect.} = 935 \\ \hline \end{array}$$

$5408 : 180 :: 400 : 14^{\circ}, 30'$

Theſe ſix Caſes may likewiſe be applied to all Parts of practical Mathematicks.

CHAP. IV.

Of right angled ſpherical TRIANGLES.

TO proceed and give ſome Caſes in ſpherical Trigonometry; but,

1. It will be neceſſary to premiſe ſome Remarks about the Nature of ſpherical Angles; the four Angles made by two great Circles cutting one another, are not equal to four right plain Angles, as being ſolid Angles, by 21. *Euc.* 11. and in *Theorem* 2. of 32. *Euc.* 1. he demonſtrates that all the external Angles of all Polygons are equal to four right Angles.

Hence, ſuppoſe a Circle to be a Poligon of infinite Sides, the external Angles are equal to four right ones; but as the Sides are ſuppos'd infinite, 'tis impoſſible to aſſign an infinite Part of four right Angles; hence the Angle made by a Tangent, and the Circumference of a Circle, is leſs than any aſſignable acute Angle, by 16. *Euc.* 3. that in ſpherical Trigonometry, we can only come infinitely near the Truth.

2. In ſpherical Trigonometry the Sides are in the ſame Proportion to the Angles, as in plain; and all ſpherical Triangles,

gles, right and oblique, may be ſolv'd by the three Propoſitions of *Euclid*, mentioned in plain Triangles. To proceed.

Aſtronomical Problems, being ſome uſual Caſes of right angled ſpherical Triangles.

CASE I.

Given the Sun's Longitude and the greateſt Declination, required the preſent Declination.

Let the Longitude be 60 (*Fig.* 14. *Pl.* 4.) and the greateſt Declination 23, 30.

Square the greateſt Declination = 552, and by Proportion, 90 : 552 :: 60 : 368, that is, the Quadrant is to 60 as the Square of the whole Declination is to the Square of the preſent, the Square Root is 19, 2.

CASE II.

Given the Height of the Pole, and the Complement of the Sun's Amplitude, required the Time from Midnight or the Angle at the Pole.

Let the Height be 51° 30′
The Complement 56

Square the two Numbers 51,30 = 2652 and 56 = 3136, which added together, are = 5788, and by Caſe 5. of plain Rectangles, and Proportion 5788 : 90 :: 3136 : 49 = $3^h\ 16^m$.

CASE III.

Given the ſame Data, required the Complement of the Sun's Declination.

Find the Angle at the Pole by the former Caſe, and by Proportion 49 : 3136 :: 90 : 5741, the Square Root is 75 +, the Complement is 15.

CASE IV.

Given the Complement of the Sun's Declination, and the Complement of the Amplitude, to find the Latitude.

The Sun's Decl.	75
Com. Ampli.	56

Find the Angle at the Pole, by Caſe 2, = 49 by ſquaring 75 = 5741, and 56 = 3136, then by Proportion 49 : 3136 :: 41 : 2627, the Square Root is 51, 30.

CASE V.

Given the Poles Height and the Complement of the Sun's Declination, required the Complement of his Amplitude.

Let the Lat. be =51 30
C. Dec. =75

Square the two Numbers 51,30=2652 and 75=5655, and by Proportion 5655:90::2652:42+.

CASE VI.

Given the Latitude, the Sun being in the Equator, and two Hours from Noon, required the Sun's Altitude.

Let the Lat. be =51 30
Two Hours =30

By Caſe 5 of plain Rectangles, find an Angle by ſquaring the two Sides 51, 30=2652 and 30=900, which added together=3552, and by Proportion 3552:90°::900:23°; then from the Angle to find the Hypothenuſe 23:900:: 90:3522, the Square Root is 59, and the Complement of Altitude is 31.

CASE VII.

Given the Pole's Height, and the Sun's Declination, required the Altitude at the Hour of 6.

Pole's Height =51 30 Sq.=2652
Declination =20

Subtracted from 51, 30 leaves 31, 30 ſq. =992 the two Squares of the 6 Hours and Latitude added, =9092, and by Proportion 9092 : 90 :: 992 : 9, which gives the vertical Angle; then 90 : 400 :: 81 : 360 the Square Root is 18, 36.

CASE VIII.

Given the Co-declination, Co-latitude and the Sun, being due *Eaſt* or *Weſt*, required the Hour from Noon.

Co-declination = 70 ſq. = 4900
Co-latitude = 38,30 ſq. = 1482

By Proportion 4900 : 90 :: 1482 : 27 the Complement is 63 = 4^h 12^m.

CASE IX.

Given the ſame Data, required the Altitude, when he is due *Eaſt* or *Weſt*.

Find the Angle by the laſt, and by Proportion 90 : 4900 :: 63 : 3430, the Root is 59, and the Complement 31.

CASE X.

Given the Co-latitude and Co-declination, required the Sun's Azimuth at the Hour of 6.

Co-lat. = 38, 30 ſq. = 1482
Co-decl. = 70 ſq. = 4900
Add the two Squares = 6382

Then find an Angle by Proportion 6382 : 90 :: 1482 : 21, which added to

90, makes the Azimuth from the *North* = 111.

There are generally 16 Caſes in right angled ſpherical Triangles, but the others may eaſily be perform'd by the ſame Mothod.

CHAP. V.

Of oblique and ſpherical TRIANGLES.

AND now to proceed, and give four of the moſt uſeful Caſes in oblique angled ſpherical Triangles.

CASE I.

Given the Sun's Co-altitude, Co-latitude and the Time from Noon, required the Angle, which the Sun's Azimuth makes with his Circle of Declination.

Co-alt. = 46 ſq. = 2116
Co-lat. = 38 30 ſq. = 1482
Time from Noon = 2 Hours or 30°.

Here are two Sides and one Angle given, and by Caſe 1. of oblique plain Triangles ſquare the two Sides, and by Proportion 2116 : 30 :: 1482 : 21.

CASE

CASE II.

Given the Co-latitude, and the two acute Angles, required the Sun's Co-altitude.

Co-lat. = 38,30 ſq. = 1482
Ang. 1 = 30
2 = 21

By Proportion 21 : 1482 :: 30 : 2116, the Square Root is 46.

CASE III.

Given the Complement of the Pole's Elevation, the Sun's Co-altitude and one Angle, required the Complement of the Sun's Declination.

Co-latit. = 38, 30 ſq. = 1482
Co-altit. = 46 ſq. = 2116

The Angle = 21.

By Caſe 2. of plain oblique Triangles, find the other Angle by Proportion 1482 : 21 :: 2116 : 30, then adding the two Squares = 3598, the other Angle = 129, 129 : 3598 :: 180 : 5020, the Square Root is 70.

CASE IV.

Given the three Sides of an oblique angled ſpherical Triangle, required an Angle: Here the two Rectangles come in to be added in obtuſe angled ſpherical

Triangles, and ſubtracted in acute ones; let the three Sides be the Complement of the Latitude, the Co-altitude and Co-declination.

Co-latit. = 38, 30 ſq. = 1482
Co-alt. = 46 ſq. = 2116
Co-declin. =70 ſq. = 4900

To find the Angle at the Pole, add the two leaſt Squares, which ſubtracted from the Biggeſt, leave the two Rectangles = 1302, which added together with the Squares, = 9800; then by Proportion the Squares of all the Sides and Rectangles are to all the Angles, as the Square of the Co-altitude is to the Angle at the Pole, or 9800 : 180 :: 2116 : 39.

And the ſame Method will work an acute angled Triangle, only ſubtracting the Rectangles from the Squares of the Sides.

3. And here may come in a very uſeful Problem at Sea, which is to find the Latitude at any Time, by two Obſervations of the Sun or Stars, and knowing the Difference of Time, together with their Declinations; for here are three Sides of a ſpherical Triangle given, to find the obtuſe Angle by Caſe 6. of oblique angled Triangles; and from

hence to find the Latitude, by Caſe 7. of plain Rectangles.

4. As likewiſe an univerſal and uſeful Propoſition or Proportion, to find the Azimuth, or riſing and ſetting of the Sun or Stars, by having the Latitude of the Place, and their Declination given; when the Sun is in the Equator it riſes and ſets univerſally at Six a Clock, unleſs directly under the Poles, becauſe half the Equator is always above the Horizon and half under, and the Line of Declination and the Equator make a Triangle ſimilar to a Triangle, made by the Latitude and Co-latitude on the Horizon, as is plain on the Globe; wherefore by Proportion as the Co-latitude is to the Declination, ſo is the Latitude to the aſcenſional Difference from the Equator or Hour of 6, to be added to 6 in *South*, and ſubtracted in *North* Declination: For Inſtance, let the Declination of the Sun be 20, by Proportion $38^\circ, 30' : 20^\circ :: 51^\circ, 30' : 26^\circ, 30'$ which turned into Time, is equal to $1^h\ 46^m$, which added to $6 = r.\ 7^h\ 46^m\ 5^h$ ſet. And by the Declination, the Sun's Place in the Ecliptick may be found by the Reverſe of the firſt Caſe of plain ſpherical Triangles, or having the Sun's Place in the Ecliptick, 'tis eaſy to find the Declination by the

the ſame Caſe, and by the Latitude and Declination, the Crepuſculum is found: For Inſtance, for the Latitude of 51,30 by Proportion $38^{\circ},30' : 18 :: 51^{\circ},30' : 24$, which turn'd into Time $= 1^{h}\ 36^{m}$. Hence by one Meridian Obſervation, and having the Latitude of the Place or Longtude or Declination of the Sun or Stars, all theſe uſeful Propoſitions may be found in any Part of the World.

5. But then as for Primes and Seconds, *&c.* becauſe it is difficult to multiply and divide them in Sexageſimals, they may be turned into Decimals, by dividing them by 60, and Decimal Parts are turn'd into Sexageſimals, by multiplying them by 60, or the eaſier Way will be to reckon ,100 for 60 Primes ,00100 for 60 Seconds ,0000100 for 60 Thirds, which makes them Decimals, and $30', 30'', 30'''$ are equal to ,555, and ſo of any other Numbers. A Table for Primes will ſerve for Seconds and Thirds, by putting them in the ſecond or third Place, as, ſuppoſe it is 57 Seconds, put two Cyphers before, as 0095.

6. The firſt ten numeral Figures are as near as any common Inſtruments require, for the greateſt Error will not exceed three Primes or Seconds; and to

this a Table is eaſily form'd; for Inſtance,

Sex.		*Dec.*
60	=	,10
54	=	,9
48	=	,8
42	=	,7
36	=	,6
30	=	,5
24	=	,4
18	=	,3
12	=	,2
6	=	,1

That is, from 60 to 54 is ,9, from 54 to 48 is ,8, *&c.* and the greateſt Error can be but 3 Primes, by taking the upper or lower Number, when it is more or leſs than half; for Inſtance, ſuppoſe it 9 Primes, take either 2 or 1 Decimals, the Error is but 3 Primes, or a more exact Decimal Table may be made.

CHAP. VI.

The Application of TRIGONOMETRY *to* NAVIGATION.

1. BUT to proceed, and make this Part likewiſe practically uſeful, and Navigation being as neceſſary as any Part, and what is done in that may eaſily be applied to other Practices, as Surveying, *&c.* To begin therefore with Navigation.

2. But before the Subject is treated of, 'twill be neceſſary to premiſe two Things, the firſt is an eaſy Method to compute a Ship's Way without any Glaſs, *&c.* and this is done by obſerving the Beats in a common Watch, which with a little Practice may be done without a Watch.

3. There are reckoned about 17200 Beats of a Watch in an Hour, and a Statute Mile conſiſts of 1760 Yards; if the Beats are 17600, it is juſt 10 Beats to one Yard, or 17600 the Beats of an Hour to a Mile; 'tis eaſy to learn to number from a Watch, or telling 10 almoſt as faſt as one can repeat it, the Error will be little or nothing.

4. But to meaſure a Ship's Way by this Method, 'tis neceſſary either to meaſure out a certain Length, as ſuppoſe 10 Yards on the Ship's Side; or the moſt ſecure Way will be to have a Log-line of about 20 Yards, with 10 Yards mark'd upon it, and when it comes to the firſt Yard in the Hand, begin to number the Tens; if 10 Yards are run in telling one 10, the Ship's Way is 10 Mile an Hour; but if only one Yard was run, it would be one Mile an Hour; and thus in any Proportion the Miles are according to the Yards in telling 10:

One Thing ought to be taken Notice of, that as one Degree upon the Equator is equal to 70 Statute Miles, wherefore to every Yard add 1 half Foot, which makes the divided Part of the Line equal to 11 Yards 2 Foot; and this gives the true proportional Diſtances upon the Earth with the Aſtronomical Obſervations. This Way of reckoning may be done in any Weather, and by a ſingle Perſon the ſame may be done, with reſpect to walking or riding; for as many Yards as are run in telling 10, ſo many Miles will it be an Hour.

5. The ſecond Thing to be premis'd is the firſt *Corollary* 32. *Euc.* 1. where he demonſtrates that in all Polygons the internal Angles are equal to twice as many right ones except 4, as there are Sides in the Polygon, and by this in a Traverſe of proportional Parts 'tis known, if all the Angles are rightly taken, notwithſtanding the Variation of the Compaſs Lee-way, and the Currents of the Sea.

CHAP.

CHAP. VII.

To find the Longitude by the COURSE.

1. THE great Difficulty in Navigation is to find the Longitude, or Diſtance from *Eaſt* to *Weſt*. The Latitude or Diſtance from *North* to *South* is found by Obſervation, becauſe the whole Revolution of the Heavens is from *Eaſt* to *Weſt*, and the fixed Stars having always the ſame Latitude, 'tis eaſy by Inſtruments to take their Diſtance from the Zenith of Obſervation when upon the Meridian, and knowing the Declination, either *North* or *South* of the Equator, which added, or ſubtracted, gives the true Latitude of the Place, the Declination of the Sun continually varying; there are Tables which give the true Increment or Decrement for every Day.

2. But 'tis more difficult to find the Longitude by Obſervation, as there are no Bodies that move from *North* to *South*, tho' this in ſome meaſure may be compenſated by the Motion of two Bodies from the *Weſt* to the *Eaſt*, when one moves faſter than the other, as the

Moon goes round above twelve Times to one Revolution of the Sun. Hence here are two Ways of diſcovering the Longitude, either by keeping a true Traverſe, or by Obſervation.

3. If the Angle of the Courſe is known, the Longitude is found out by by the Latitude; for it is of the ſame Nature with the Angle of Parallax in which the nearer the Body is, the greater is the Angle of Parallax, and the leſſer the Angle of Elevation: For Inſtance, (*Fig.* 1. *Pl.* 2.) let *d* and *e* be two Bodies, one at a greater Diſtance than the other, the Angle *b d c* is greater than the Angle *b e c*, tho' ſeen from the Center in the ſame Line *c d e f*, and both ſubtended by the ſame Line *b c*; for *b d* and *d c* are leſs than *b e* and *e c* by Suppoſition, wherefore the Angle *d* is leſs than the Angle *e*, by 21. *Euc.* 1. and the Angle of Elevation *c b d*, is leſs than *c b e* by the ſame.

4. This is the Nature of Longitude; for ſuppoſe two Places, as *f* and *g* (*Pl.* 2. *Fig.* 2.) to be in the ſame Latitude, but of different Meridians or Longitude, the Angle of the Courſe of the greater Longitude is bigger than the Angle of the Courſe of the leſſer, for the Angle *f c d* is leſs than the Angle *g c d*, by 21. *Euc.* 1.

for

for tho' the Parallels of Latitude are always or nearly at right Angles to the Meridians, yet in a Traverſe the right Angle is only made by the firſt Meridian.

5. Hence the Longitude is found by the Courſe and Latitude; for Inſtance, let p be the Pole of the World, l the Latitude $= 51\ 30$, d the Difference of Latitude $= 5$, and $\ae q$ the Equator; let dcf or the Courſe be 50, to find df the Difference of Longitude. Here is one Side and Angle given of a Rectangle Triangle, by Caſe 1. of plain Rectangle Triangles; ſquare the Side $5 = 25$, and by Proportion $40^\circ : 25 :: 50 : 31\frac{1}{4}$ the Square Root is 5, 6; but as the Degrees are leſs than equatorial Degrees, the Difference is found by another Proportion; pdf and $p\ae h$ are ſimilar Triangles; hence $pd : p\ae :: df : \ae h$ by 4. *Euc.* 6. but pd is equal to $38^\circ\ 30' + 5 = 43\ 30$, wherefore $43, 30 : 90 :: 5 : 10, 37$ the true Meridian or Longitude of f. This is for a ſingle Courſe.

6. But when there is a Complication of Courſes, or a Traverſe, the Difficulty is greater by reaſon of the Variation of the Compaſs, Lee-way, and Currents of the Sea; the Variation of the Compaſs may one way be corrected if the Hour

of the Day or Night is known, but the Uncertainty of Latitude at Sea makes it difficult to find the Hour by Day, which at Night may be found by the *North* or *South* Pole, with a double equinoctial Dial, or three Rings one within another, which will likewiſe ſerve for an Aſtrolabe much better than the common Inſtruments; by it the Diſtance of any Star round the Pole, from the upper or lower Meridian, is given, which will determine the true Place of the Pole; and hence the Hour and Latitude is known: For Inſtance, the Pole Star is 2° 24 diſtant from the Pole, and its right Aſcenſion 9,30, the Pole lies between that and a Star in the oppoſite Meridian, the firſt in the Tail of the Bear comes neareſt, whoſe right Aſcenſion is 190° 30, and ſubtracting 9°, 24′ leaves 181°, 6′, which is 1° 6′ to the *Eaſt* of the oppoſite Meridian, that when this Star is in the upper or lower Meridian, the Declination of the Pole Star muſt be added or ſubtracted, and it gives the true Latitude; the ſame may be done by knowing the right Aſcenſion of any other Star round the *North* or *South* Pole, for by knowing the Diſtance of the Pole Star, from the upper or lower Meridian, by Proportion to find the Difference: For

Inſtance, ſuppoſe the Pole Star be 40 Degrees to the *Eaſt* or *Weſt* of the upper or lower Meridian $90:2^{\circ}, 24' :: 40^{\circ} : 1^{\circ}, 6'$, which is eithe radded or ſubtracted, according to the former Directions. By the right Aſcenſion and Diſtance of any Star from the Meridian, to know the Hour of the Night, ſubtract the right Aſcenſion of a Star, from the right Aſcenſion of the Sun, the Star's Diſtance from the Meridian will determine how far the Sun is paſs'd the Meridian: For Inſtance, ſuppoſe the firſt in the Bear's Tail be 28 Degrees to the *Eaſt* of the Meridian, and the Sun in the firſt Degree of *Gemini*, to find the Hour of the Night, ſubtract the right Aſcenſion of the Sun from the Star; or 60 from $190^{\circ}, 30'$ remains 130, 30, but becauſe the Star is not yet come to the Meridian, add 28 to 130,30, gives $158,30 = 10^{h}\ 54^{m}$; and for the Latitude, by *Prop.* $90 : 2^{\circ}, 24' :: 28 : 46$, or $1^{\circ}\ 38$ muſt be added to the Pole Star, and the Diſtance of the Star from the Meridian will ſhew the Variation of the Compaſs.

7. And now to proceed, and give a Method of working a Traverſe by proportional Parts, which with the Latitude and Corol. will determine the Longitude; one Inſtance of two Courſes will be

ſufficient. (*Fig.* 13. *Pl.* 4.) Let *abcd* be the Traverſe, *ad* the Meridian of departure *ab* and *bc* the two Courſes, the Difference of Latitude and Meridian always make a right Angle, and are two Sides of the Polygon. According to the Method of finding the Ship's Way, ſuppoſe the firſt run 12 Hours, and 1 ten told for every five proportional Parts, which is after the Rate of five Miles an Hour, this 5 multiplied by 12, gives 60 proportional Parts for *ab*, and let the Angle of the Courſe *bad* be 50°, let the other Courſe, or *cbe* be 30°, and the Run 7 Hours and 1 ten told for every 10 Knots, which is 10 Miles an Hours, which multiplied into 7, makes it 70 proportional Parts, for the ſecond Diſtance or *bc*.

To find the Angle *dac*, or the Courſe from the firſt Meridian, which determines the Longitude, by the *Corollary* try if the Angles are all taken right, or if the Lee-way Variation or Currents have cauſed any Error; in this Traverſe or Polygon; here are 4 Sides, and conſequently 4 Rectangles, for twice 4 is 8, and 4 ſubtracted leaves 4, every different Courſe has a different Meridian, which will diſcover the Angles: Hence

a =

$a = 50$
$b = 160$
$c = 60$
$d = 90$

---

360 equal to 4 Rectangles: Hence here is no Variation, &c. to be allow'd for.

8. Draw ca to find the Angle cad; here are two Sides given, ab and bc, and the Angle between them abc; for that Angle is always equal to the Complement of the firſt Angle of the Courſe, to the Angle of the ſecond Courſe and a Rectangle, becauſe the Meridian and Latitude cut one another at right Angles, wherefore the firſt Angle being 50, the Complement is 40, the ſecond Angle is 30, which with the Rectangle are equal 160, and the Sum of the other two Angles equal to 20; ſquare the two Sides ab 60 = 360 and bc 70 = 4900, adding them together = 8500, by Caſe 4. of oblique plain Triangles 8500: 20:: 4900: 11°,39′, which is the Angle at a, which ſubtracted from 50 leaves 38° 21′, for cad the Angle of the Courſe, whence 'tis eaſy to find the Longitude by the firſt Caſe; and if there be an Error committed in the Diſtances or Courſes they

are corrected by the Corollary and Latitude.

9. The Corollary corrects the Angles and the Latitude; both the Angles and Diſtances; for if *ad*, the Difference of Latitude, is the ſame in Caculation and Obſervation, the Traverſe is true, and conſequently the Longitude eaſily found; find therefore *ac*, by Caſe 5. of oblique plain Triangles, by adding the Squares of the two Sides = 8500, and by Proportion 160°: 8500 :: 180 : 9562 the Square Root is 98, from whence 'tis eaſy to find *ad* the Latitude, or *cd* the Longitude, to find the Difference of Latitude *ad*, by Caſe 7. of plain Triangles, 90 : 9562 :: 52 : 5524, the Square Root is 74, the Longitude is found by the other Angle; and if the Obſervation anſwers to this Latitude, the Diſtances and Angles were taken right; for the Angles of all Polygons of an equal Number of Sides are equal, and conſequently there can be no Error in the Angle or Diſtance, but what is proportional by making a Polygon greater or leſs and ſimilar to another, but this is impoſſible in this Caſe, becauſe here is one Side of the Polygon always given, which is the Difference of Latitude, the equatorial Degrees are

found out, as in Caſe 1. of a ſingle Courſe. And

10. By this Method may a Traverſe of never ſo many Courſes be work'd; for Inſtance, (*Fig.* 3. *Pl.* 2.) ſuppoſe a Traverſe of 4 Courſes, find the two firſt Courſes *a d* and *d e*, and the Side that ſubtends them *a e* then the other Courſes *e f*, *f c*, and the Side that ſubtends them *e c*, and draw *a c*, which is found by *a e* and *e c*, and conſequently the Angle *b a c* which determines the Longitude; the ſame Method is taken if it ſhould paſs and repaſs the firſt Meridian. Theſe Triangles may be wrought by Logarithms, but they are more tedious and uncertain.

CHAP. VIII.

To find the LONGITUDE *by Obſervation.*

THE other Way of finding the Longitude is by Obſervation; but this requires greater Accuracy both in Obſervation and Calculation, and it is done either by a perpetual Motion, or the Receſſion of the Moon and Satellites of *Jupiter*.

1. If a regular even Motion could be found that would meaſure out equal Time, in order to make it uſeful at Sea to find the Longitude, it muſt be fix'd to the Hour of the Meridian of Departure; and if the Time be taken at Sea, the Difference of Meridians or Longitude is eaſily known, but the Difficulty is to find the true Time; for if there be an Error of 4 Minutes, it is equal to a whole Degree of Longitude; and ſuch Errors are eaſily committed where the Ship is always in Motion, and the Latitude doubtful; for the Meridian Height of the Sun can ſcarce be diſcovered at Sea within three or four Degrees of Longitude, the riſing and falling at that Time is ſo inconſiderable.

2. If it is done by the Satellites of *Jupiter*, it is liable to the ſame Uncertainties of Time, beſides the Impoſſibility of ſeeing them without Teleſcopes, which are uſeleſs at Sea; or ſuppoſe it done by the Moon's Receſſion, or going from the Sun, which is about 32 Primes in an Hour, equal to 15 Equatorial Degrees; but as the Degrees are leſs in Proportion to the Diſtance of the Obſerver from the Equator, the Error is leſs, for in the Latitude of 45: 32 Primes of Motion make but 7° 30' Equatorial Degrees, and

in our Latitude about 6°; but if the Moon's Place is found to 2′ the Error of Longitude is about 20 Miles, ſo that in the *Northern* and *Southern* Seas it may be of great uſe, eſpecially in long Voyages, or when the Angle of the Courſe is loſt.

3. There are two Ways of knowing the Longitude by the Moon, the Eclipſes or Change and Receſſion; for ſuppoſe an Eclipſe happen at 8 a Clock in the Morning in this Meridian, and the ſame to happen at 12 in another, the Difference of Meridians is 4 Hours in Time, which turn'd into Degrees are equal to 60°, the Difference of Longitude or Meridians; or it may be known by other Changes of the Moon, where the true Account of theſe Changes is kept, for this gives the Difference of Time: For Inſtance, ſuppoſe it New Moon at 12 a Clock at *Pekin* in *China*, and 7^h 20^m after at *London*, multiply the Time by 15° and it gives 110° to the *Eaſt* of the Meridian of *London*; this may be done by the common Almanack of each Country, where the true Account of the Moon's Changes are kept; but this is not practicable at Sea.

The other Way is by the Receſſion of the Moon; and this is done by Calculation

culation and Obſervation; and this may be us'd at Sea, for the Difference of Calculation and Obſervation gives the Difference of Meridians; but in order to make it more clear, it will be neceſſary to premiſe ſomething about the Correction of Time, as well as how to make an Obſervation.

CHAP. IX.

The Correction of TIME.

1. TIME is meaſured by the Motion of the Sun, or rather by the Motion of the Earth round the Sun, of which our Revolution is called a Year, and another Motion of the Earth is round its own Axis in 24 Hours, which meaſures out Night and Day. Every Country had a different Way of reckoning the Year, the *Jews* by the Number of Moons, and the firſt regular Settlement of the Solar Year was by *Julius Cæſar*, which was 45 Years before the Birth of our Bleſſed Saviour; for the preceeding one, which was called a Year of Confuſion, conſiſted of 445 Days, which he reduced to 365 Days and 6

Hours, which 6 Hours in 4 Years makes a Day, and added to the Month of *February* makes it to conſiſt of 29 Days, and it is called *Leap Year*, or *Biſſextile*, becauſe in every fourth Year the ſixth of the Kalends of *March* was accounted twice; but this 6 Hours, according to the Tables of the Sun's Motion, are too much.

2. The Year begins the firſt of *January*, and ends the 31ſt of *December*, for in that Time the Sun goes nearly thro' all the twelve Signs; but the Longitude of the Sun the 31ſt of *December* is 11^s, 29°, 45′, 40″, which ſubtracted from 12 Signs in order to begin the Year, leaves 14 Primes, 20 Seconds too little; hence more time muſt be added in the Tables of Hours, 14 Primes, 47 Seconds are equal to 6 Hours, but 47 Seconds are too much; ſubtract therefore 20, and remain 27 Seconds, and 27 Seconds and 6 Thirds are equal to 11 Minutes, which is too much by 6 Thirds, which is equal to 2 Seconds, 31 Thirds of Time, which ſubtracted from 11 Minutes remain 10^m, 57 Seconds, 29 Thirds, and this ſubtracted from 6 Hours leaves 5^h 49^m 2^s 31^t of Time, which is equal to the 14 Primes, 20 Seconds of Motion; hence the whole Year conſiſts of 365^d

$5^{h}\ 49^{m}\ 2^{ſ}\ 31^{t}$. The Syderial Year is one whole Day or 24 Hours leſs, wanting about 3 Seconds of Time to be allowed for the Proceſſion of the Equinox; for a Star in the Meridian in 24 Hours Time, or in one Revolution of the Sun or Earth will, be paſs'd the Meridian 59 Primes, 8 Seconds, which is the Receſſion of the Sun in 24 Hours, and in one Year makes a Day.

3. The Seaſons therefore, ſince the Time of *Julius Cæſar*, muſt have receded, by reaſon of the odd Minutes which are not taken Notice of in his Account of Time, and to find out how much, add 45, the Year of Correction, to 1735 the preſent Year, which makes 1780, and multiply this by the Minutes and Seconds, and it ſhews how many Days the Seaſons have receded; bring the Minutes into Seconds, which are 657, and multiplied by 1780, are equal to 1169460, and divided by 60, to bring them again into Minutes, are equal to 19491, which divided by 1440 the Minutes of a Day, gives 13 Days, 12 Hours, 51 Minutes, that the Seaſons have receded ſince *Julius Cæſar*'s Time; and this is the Cauſe of the old and new Stile, for in *Catholick* Countries they reckon 11 Days before us, and it is no

more, or rather leſs, becauſe they reckon from the firſt Council of *Nice*, which was in 325, which added to 45 is $=370$, and divided by $131 = 2^{d}\ 19^{h}$, which ſubtract from 13^{d}, 12^{h}, 51^{m} leaves 10^{d}, 17^{h}, 51^{m}; 6^{h}, 9^{m}, too little, that this Account is not exactly true.

4. But to find out in what Year another Day recedes, by Proportion, the Seconds of one Year are to one Year, as the Seconds of a Day are to a 4th Number, or $657^{\text{ſ}}.: 1^{y} :: 86400^{\text{ſ}}.: 131\frac{333}{657}$; hence in every 131 Years a Day recedes. For the Year find the Year when the odd Hours and Minutes make a Day, by Proportion $86400 : 131 :: 46260 : 70$, which added to $1780 = 1850$ the Year, the Seaſons will recede 14 Days, and every 131 Years after; as for the odd Seconds that are more than 131, and what remains over 70, which in about 33929 Years makes a Day; but by that Time I ſuppoſe we ſhall have no Occaſion for it, ſo that 131 will preſerve the Seaſons as they were in *Julius Cæſar*'s Time, to a ſufficient Exactneſs.

5. The neglecting theſe odd Minutes is the Cauſe of the many Errors which have been committed in keeping the moveable Feaſts; for had the firſt Reformers been as careful in correcting the

Calendar, as they were induſtrious in reforming the antient Religion, they might have avoided thoſe Miſtakes; for inſtead of the 21ſt of *March*, their ſtanding Rule in the *Common Prayer Book*, they ought, according our Time, or old Stile, to have put down the Equinox, or 10th of *March*, which is nearer, and the Rules would not have erred: For, to find the Moon's Age for any Time, find the *Golden Number*, by adding 1 to the Year, and dividing by 19 and taking the Remainder; from hence the Epact is found by dividing the *Golden Number* by 3, and adding 10 for each one, which add to the Epact and caſting out the 30ſ. then for the Moon's Age in any Month, add the Day of the Month, Epact, and the Months incluſive from *March*, it gives the Moon's Age: For Inſtance, 1736 to which add 1 = 1737, and divided by 19, the Remainder is 8, the *Golden Number*, which divided by 3 remains 2, and 10 Times 2 = 20, added to 8 = 28 the Epact, becauſe under 30. Required the Moon's Age 25th of *April*, the Days, Epact, and Months = 55, from which ſubtract 29, becauſe the Month hath but 30 Days, remain 26 the Moon's Age, and conſequently the New Moon was the 31ſt of *March*, or according to the new Stile, the 10th of *April*, and the other New Moon was the 1ſt or 11th of *March*, about the Equinox, which was the true Paſcal Moon, according to true Time; for the Reaſon why the Council of *Nice* fix'd the Paſcal Moon to the 21ſt of *March*, was, becauſe at that Time it was about the Equinox; the Rule of the moveable Feaſts, which are govern'd by the Moon, as the fix'd ones are by the Sun, was firſt given to the *Jews* in the Celebration of their Paſca or Paſſover, which happened then to be the 14th Day of the firſt Moon or Month after the Equinox, which anſwers to our *March*; and the Chriſtians, becauſe they would not keep their Paſca on the ſame Day with the *Jews*, appointed that *Eaſter* ſhould always be celebrated on a Sunday, and if the Paſcal full Moon ſhould happen on a Sunday, the next Sunday after to be kept: But we neither keep our *Eaſter* according to the *Chriſtian* or *Jewiſh* Rule, but have our *Eaſter* to ourſelves; true *Engliſhmen!* for in 1735 the full Moon was the 27th of *March* on a Thurſday, but *Eaſter* was kept the Sunday ſeven-night after the 6th of *April*, which ſhould have been kept the next Sunday, or 30th of *March*; the ſame Error happens this Year in 1736 the full Moon is on *Wedneſday* 14, but the *Eaſter* is kept on the 25th, a Week too late; beſides the Month, for want of true Time, and for want of taking Notice of the odd Minutes, we have, ſince the Paſſover was firſt inſtituted, almoſt loſt the Paſcal Moon; for from 45 Years before the Birth of our Bleſſed Saviour, to the firſt Inſtitution of the Paſca, is about 1486 Years, which divided by 131, leaves $11^d\ 8^h\ 14^m$, which added to $13^d\ 12^h\ 51^m = 24^d\ 21^h\ 05^m$; and if this be carried up to the Creation of the World, which was about 3983 Years before the Birth of our Bleſſed Saviour, from whence ſubtract 45, leaves 3938, and divided by 131, leaves $30^d\ 1^h\ 28^m$, which added to $13^d\ 12^h\ 51^m = 43^d\ 14^h\ 19^m$; that if the firſt of *March* was the Time of the Equinox, and the Time when the World was firſt created, it muſt now, according to our Time, be the 22d of *April*; and this is as proper a Time as any to fix the Equinox or 10th of *March* in: Or ſuppoſe the 11th of *December*, when the Sun enters into *Capricorn* to be the 25th, this would bring back the Seaſons to *Julius Cæſar*'s Time, and the Nativity of our Bleſſed Saviour would fall on the ſhorteſt Day of the Year, as it really did, and the Sun enter into *Aries* the 23^{d} of *March*, which by means of the intercalary Days may be kept to a Minute, for by reaſon of the odd Minutes, we have loſt, ſince the Creation, almoſt a Moon and half; and this Difference of Time ought to be particularly taken notice of in making the Tables of Motion; for if the Time be not rightly adjuſted 'tis impoſſible to make any true Obſervations; and 'tis probable this is one principal Cauſe of thoſe Intricacies, which are found in the practical Part of Aſtronomy. So much for the Correction of Time.

CHAP. X.

To make Obſervations of the LONGITUDE.

1. AS for making Obſervations: The Latitude is eaſily taken either by the Sun or Stars when upon the Meridian, by knowing the Declination, and adding or ſubtracting, as it is *North* or *South* of the Equator.

But for the Longitude, many Methods are propos'd; but the moſt practical for all Times ſeems to be by taking the

Diſtance or Longitude of the Sun or Stars from the Meridian of Obſervation, which is done by taking the Height and knowing the Declination, together with the Latitude of the Place; for in this Caſe there are three Sides of an oblique ſpherical Triangle given, the Complement of Latitude, the Complement of Altitude, and the Complement of Declination, to find the Angle of the Pole, which is done by Caſe 4. of oblique ſpherical Triangles; or it may be done by the Rectangle, when the Sun or Star is to the *South* of the *Eaſt* or *Weſt* vertical Circle, as by the Difference between the Declination and Latitude, together with the Diſtance from the Zenith, and this is done by Caſe 3. of plain Rectangle Triangles: For Inſtance, (*Fig.* 12. *Pl.* 4.) let *a c q* be the obtuſe angled Triangle, in which are given all the three Sides, to find the Angle *c a q*, and let *e q æ* be the Rectangle, to find the Angle *q c æ*, which by Caſe 1. of plain Rectangle Triangles will find the Side *q æ*, only allowing for the Difference of Equatorial and other Degrees, and by this means may the Diſtance of any two Stars be taken; for if the Stars are both of the ſame Side of the Meridian, ſubtract the leaſt Diſtance from the greateſt, and it gives the Diſtance from one another; or if they are of both Sides the Meridian, add them together, and it gives the ſame.

2. If the Diſtance of a Star from the Meridian be taken, whoſe right Aſcenſion and Declination are known, and the Sun's right Aſcenſion likewiſe known, 'tis eaſy to determine the Hour of the Night, as mentioned in *Chap.* 6. *Sect.* 6. P. 2. and by this means may the Longitude of the Moon be found, by taking the Diſtance from a Star, and that from the Sun; and if the Calculation and Obſervation of the Moon anſwer, 'tis plain the Tables of the Moon were made for that Meridian, but if they vary, the Difference ſhews the Difference of Meridians: For Inſtance, ſuppoſe the Moon's Place is calculated at Sea for 6 a Clock, by Tables that are made for the Meridian of *London*, but by Obſervation it does not come to that Place till 8 a Clock, which is about 4 Degrees Receſſion from the Sun; for the Moon in a mean Number recedes about 30 Primes an Hour from the Sun, and from this Obſervation, 'tis plain the Meridian is to the *Weſt* of *London*, and the Difference is found by Proportion; for ſuppoſe the Moon recedes 12 Degrees in going 360, then 12 : 360 :: 4 : 120

which is the Difference of Meridians; or the Receſſion of the Moon may be found by the Latitude upon the Meridian, becauſe it alters the Latitude as it does the Longitude; but Obſervations this way cannot ſo frequently be made.

CHAP. XI.

To find the Moon's *Place.*

HOW to find the Moon's Place by Calculation, ſo as to anſwer to the Obſervation.

1. There are two Things to be obſerv'd in calculating the Moon's Place.

The Ecliptick, which is always the ſame, and anſwers to the Calculation of the mean periodical Motion, which is in 27^d 7^h 43^m 7^s; for ſuppoſe, as in the Tables, the Moon's Longitude is

Jan. 27.	11	25	45	45
7^h		3	50	35
43^m			23	36
7^s				4
	12	00	00	00

= to the periodical Revolution. And the Line of the Moon which cuts the Ecliptick nearly in two oppoſite Points called the Nodes; and this is the Line of Obſervation, or where the Moon revolves in the ſame Time: Hence when the Place of the Moon is calculated, it does not anſwer in Obſervation, either as to Longitude or Latitude; the Longitude and right Aſcenſion are the ſame, as the Circles of the Ecliptick and Equator are equal, only the Degrees of the Ecliptick, as nearer to the Pole, are leſſer.

2. But to make it clearer, 'tis neceſſary to premiſe ſomething about the Nature of the Moon's Line; for Inſtance, (*Fig.* 6. *Pl.* 2.) let *æq* be the Equator, *ec* the Ecliptick, and *lm* the Line of the Moon; the Form of this Line continually varies by reaſon of the Nodes receding about 1° 22′ every periodical Revolution, which Receſſion of the Node is owing to the Difference of the Moon's Latitude; for the greater Latitude, according to *Tycho*, is 5°, 17′, 30″, and the leaſt is 4° 57′, the Difference is 20′, 30″, which multiplied by 4, becauſe it muſt affect the Node four times, for as it ſhortens twice in one Semicircle, it lengthens twice in the other, which is equal to 1° 22′ equal to the Receſſion of the Nodes in a periodical Revolution.

3. The Circle of the Moon and Ecliptick are equal, but have a different Center, and the Diſtance is equal to the Difference of Latitude, the Circles are to be conſidered as on a Sphere and not a Plane. Let therefore (*Fig.* 8. *Pl.* 2.) *l d m* be the upper Semicircle, and *e a c* the under one, and *a d* equal to the Difference of their Centers, 'tis plain *b g* is equal to *d a*, and that if *l d m* were continued to the Diameter of the under Semicircle, it requires the Difference of Latitude to be added to each End, but if it be a Quadrant, once the Latitude; (*Fig.* 9. *Pl.* 2.) for the two Semicircles *a e d* and *c e b* are equal; but the altering the Centers ſo many Degrees as the Difference of Latitude is, puts it in this Form, and this is the Nature of the Lunar Line, as well as of the Planets that have a Node. Hence 'tis plain from Figure 1. that the Moon is ſometimes too faſt and ſometimes too ſlow, for the Ecliptick, as it is farther or nearer the Equator, and as the Lunar and Ecliptick Lines make two Sides of a Triangle, the Velocity and Slowneſs are in Proportion to the Latitude and Diſtance from the Node; and this, with the Anomaly of Equation and Velocity, will account for the Irregularities in the Moon's Motion.

4. Suppoſe therefore the Moon paſt the *Southern* Node the 29th of *November* 1735, the Node is $6^{\text{ſ}}$, 12^{d}, $9'$ $27''$, and the Moon $6^{\text{ſ}}$, 27^{d}, $32'$, $59''$ the Difference is 15^{d} $23'$ $32''$ that the Moon is paſs'd the Node, and becauſe the Lunar Line is nearer the Equator, conſequently the Moon moves too faſt for the Calculation, and as the Difference of Velocity is in Proportion to the Difference of Latitude; wherefore $90^{d} : 5^{d}, 17', 30'' :: 15^{d}, 23', 32 : 54$, about 54 Primes before the Calculation; and thus always as the Latitude between the Lunar Line and Eccliptick increaſes, the Velocity of the Moon's Motion increaſes, and when the Latitude decreaſes, the Velocity likewiſe decreaſes, as is plain from *Chap.* 3. *Sect.* 5. of the firſt Part. A Table of the Moon's greater and leſſer Latitude, inſerted for every Degree.

A

A TABLE of the Moon's Latitude, according to the Lunar Line and Diſtance.

From the Node. For the 1ſt Node.

Deg.	Lat.			Deg.	Lat.			Deg.	Lat.		
	d	′	″		d	′	″		d	′	″
1	0	3	30	31	1	48	30	61	3	35	30
2	0	7	00	32	1	54	00	62	3	39	00
3	0	10	31	33	1	57	41	63	3	42	51
4	0	14	00	34	2	1	0	64	3	46	00
5	0	17	30	35	2	4	30	65	3	49	30
6	0	21	02	36	2	8	12	66	3	53	22
7	0	24	30	37	2	11	30	67	3	56	30
8	0	28	00	38	2	15	00	68	4	00	00
9	0	31	33	39	2	18	43	69	4	3	53
10	0	35	00	40	2	22	00	70	4	7	00
11	0	38	30	41	2	25	30	71	4	10	30
12	0	42	04	42	2	29	14	72	4	14	24
13	0	45	30	43	2	32	30	73	4	17	30
14	0	49	00	44	2	36	00	74	4	21	00
15	0	52	35	45	2	39	45	75	4	24	55
16	0	56	00	46	2	43	00	76	4	28	00
17	0	59	30	47	2	46	30	77	4	31	30
18	1	3	06	48	2	50	16	78	4	35	26
19	1	6	30	49	2	53	30	79	4	38	30
20	1	10	00	50	2	57	00	80	4	42	00
21	1	13	37	51	3	00	47	81	4	45	57
22	1	17	00	52	3	4	00	82	4	49	00
23	1	20	30	53	3	7	30	83	4	52	30
24	1	24	08	54	3	11	18	84	4	56	28
25	1	27	30	55	3	14	30	85	4	59	30
26	1	31	00	56	3	18	00	86	5	30	00
27	1	34	39	57	3	21	49	87	5	6	59
28	1	38	00	58	3	25	00	88	5	10	00
29	1	41	30	59	3	28	30	89	5	13	30
30	1	45	10	60	3	32	00	90	5	17	30

For the leſſer Latitude or ſecond Node.

Deg.	Lat.			Deg.	Lat.			Deg.	Lat.		
	d	′	″		d	′	″		d	′	″
1	0	3	20	31	1	42	20	61	3	23	20
2	0	6	40	32	1	45	40	62	3	26	40
3	0	10	00	33	1	49	00	63	3	30	00
4	0	13	20	34	1	52	20	64	3	33	20
5	0	16	40	35	1	55	40	65	3	36	40
6	0	20	00	36	1	59	00	66	3	40	00
7	0	23	20	37	2	3	20	67	3	43	20
8	0	26	40	38	2	6	40	68	3	46	40
9	0	30	00	39	2	10	00	69	3	50	00
10	0	33	20	40	2	13	20	70	3	53	20
11	0	36	40	41	2	16	40	71	3	56	40
12	0	40	00	42	2	20	00	72	4	00	00
13	0	43	20	43	2	23	20	73	4	3	20
14	0	46	40	44	2	26	40	74	4	6	40
15	0	49	00	45	2	30	00	75	4	10	00
16	0	52	20	46	2	33	20	76	4	13	20
17	0	55	40	47	2	36	40	77	4	16	40
18	0	59	00	48	2	40	00	78	4	20	00
19	1	2	20	49	2	43	20	79	4	23	20
20	1	5	40	50	2	46	40	80	4	26	40
21	1	9	00	51	2	50	00	81	4	30	00
22	1	12	20	52	2	53	20	82	4	33	20
23	1	15	40	53	2	56	40	83	4	36	40
24	1	19	00	54	3	00	00	84	4	40	00
25	1	22	20	55	3	3	20	85	4	43	20
26	1	25	40	56	3	6	40	86	4	46	40
27	1	29	00	57	3	10	00	87	4	50	00
28	1	32	20	58	3	13	20	88	4	53	20
29	1	35	40	59	3	16	40	89	4	56	00
30	1	39	00	60	3	20	00	90	4	57	00

5. But now to proceed, and give Inſtances of Calculation, for the four Quarters of the Moon, according to this Method of the Lunar Line, and the Anomalies mentioned in *Part* 1. *Chap*. 3. let the firſt be *November* 29, 1735.

	Longitude of the Moon				*The Node.*			
1735	4	19	48	37	6	29	47	30
Nov. 29	2	7	44	22		17	38	3
	6	27,	32,	59	6	12	09	27
Lun.Line			52	35				
	6	28	25	34				
Equat.		1	20					
	6	27	05	34				
Veloc.		8	30					
	7	05	35	34	13	12	44	
					1	34	39	
					11	38	05	

But to give an Explication of it in Words, the Node is a little more than 12 Degrees to the *Eaſt* of *Libra*, and the Moon 27, and as it moves between the Equator and Ecliptick the Velocity increaſes, which by the Tables is equal to 52, 35, and is added to the Calculation, by the Equation Table the Moon is ſlower, which is ſubtracted, and the Velocity ſlower, wherefore the Remainder is added, to which find the Declination and ſubtract the laſt Latitude, becauſe the Lunar Line is ſo much nearer to the Equator, and by the Declination and Altitude, the meridian Diſtance of the Moon is found.

6 In the next Quarter it requires a different Calculation: For Inſtance,

	The Moon's Longitude				*The Node.*			
	s	d	'	"	s	d	'	"
1735	4	19	48	37	6	29	47	30
Dec. 5	4	26	47	52		17	57	7
	9	16	36	29	6	11	50	27
Lun. Line		5	3					
	9	21	39	29				
Equa.		4						
Vel.		3	50					
	9	29	29	29	23	28	46	
					4	11	0	
					19	17	47	*Declin.*

As the Latitude of the Lunar Line is paſs'd the Quadrant, and decreaſing add the Remainder, the Equation faſter and the Remainder of the Velocity to be added, and the greateſt Latitude to be ſubtracted from the Declination.

7. The third Quarter is near the ſame as the firſt, only the Receſſion of the Node: For Inſtance, *Decem.* 12, 1735.

	The Moon's Longitude				*Moon's Node.*			
	s	d	'	"	s	d	'	"
1735	4,	19,	48,	37	6	29,	47,	30
Dec. 12	7	29	1	57		18	19	21
	0	18	50	34	6	11	28	09
Lun. Line			24	30				
	0	19	14	04				
Equa.		1						
Veloc.		1						
	0	21	14	04	8	12	36	
						35		
					7	37	36	

Here the Lunar Line is faſter than the Ecliptick, as likewiſe the Equation and Velocity, which muſt all be added, the greateſt Latitude ſubtracted for the Declination.

8. In the fourth and laſt Quarter the Moon is nearly found, as in the ſecond: For Inſtance, *Decem.* 20. 1735.

	The Moon's Longitude.				*Moon's Node.*			
	s	d	'	"	s	d	'	"
1735	4	19	48	37	6	29	47	30
Dec. 20	11	14	26	37		18	44	47
	4	04	15	14	6	11	02	43
Lun.Line		3	29	00				
	4	7	44	14				
Equa.		3						
	4	10	44	14				
Veloc.		7						
	4	17	44	14		22	24	36
						3	45	
						18	39	36

As the Moon got more in Velocity the laſt Quarter, than it loſt in this, the Difference is to be added, the Equation and Velocity added, and the greateſt Latitude to be ſubtracted from the Declination.

I ſhall put down one Obſervation as it was made, *Decem.* 18. 1735. 9ʰ at Night.

Moon's

	Moon's Longitude.	*Moon's Node.*
1735	4, 19, 48, 37	6, 29, 47, 30
Dec. 17	10, 4, 54, 52	18 35 15
H. 21	11, 31, 46	
	3, 06, 15, 15	6, 11, 12, 17
Lun. Line	4, 45, 30	the Moon's Diſ-
		tance from the
	3, 11, 00, 45	Meridian $35^{d}, 30$
Equa.	4, 40	$9^{h} = 135$
		35 30
	3, 15, 40, 45	
Veloc.	5, 20	170, 30
	3, 21, 00, 45	19, 28. *Decl.*

	The Sun's Longitude.	the Difference
1735	9, 20, 29, 41	between the
Dec. 18	11, 16, 56, 52	Obſervation &
H. 9	22, 11	Calculation, but
		2 Primes.
	9 07 48 34	
Equa.	1 52	
Veloc.	1 52	
	9, 11, 32, 34	
	3 21 00	
Diff.	5 20 32 = $170^{d}, 32'$	

And many more Obſervations have been made, tho' not always ſo accurate.

CHAP.

CHAP. XII.

TABLES *of the* SUN *and* MOON.

HERE are inſerted ſhort Tables of the mean Motions of the Sun and Moon, to be taken from the preceding Midnight, allowing for the odd Hours, the 24th of *February* or the 6th of the Calends of *March*, is the Time when Leap Year takes Place.

The Sun's mean Motion.

Years of Chriſt Current.	*The Sun's Longitude.*			
	s	d	′	″
1701	9	20	43	50
1801	9	21	29	10
1901	9	22	14	30
2001	9	22	59	50

Years

The Sun's Longitude.

Years Com.	s	d	′	″
1	11	29	45	40
2	11	29	31	20
3	11	29	17	00
4	00	00	01	49
5	11	29	47	29
6	11	29	33	09
7	11	29	18	49
8	00	00	03	37
9	11	29	49	18
10	11	29	34	58
11	11	29	20	38
12	00	00	05	26
13	11	29	51	06
14	11	29	36	47
15	11	29	22	27
16	00	00	07	15
17	11	29	52	55
18	11	29	38	35
19	11	29	24	15
20	00	00	09	04
40	00	00	18	08
60	00	00	27	12
80	00	00	36	16
100	00	00	45	20

Months	s	d	′	″
January	00	00	59	08
February	01	01	32	26
March	01	29	08	02
April	02	29	41	38
May	03	29	15	48
June	04	29	49	06
July	05	29	23	16
Auguſt	06	29	56	33
Septemb.	08	00	29	52
October	09	00	04	02
Novemb.	10	00	37	20
Decemb.	11	00	11	30

Days	s	d	′	″
1	00	00	59	08
2	00	01	58	17
3	00	02	57	25
4	00	03	56	33
5	00	04	55	42
6	00	05	54	50
7	00	06	53	58
8	00	07	53	07
9	00	08	52	15
10	00	09	51	23
11	00	10	50	31
12	00	11	49	40
13	00	12	48	48
14	00	13	47	57
15	00	14	47	05
16	00	15	46	13
17	00	16	45	22
18	00	17	44	30
19	00	18	43	38
20	00	19	42	47
21	00	20	41	55
22	00	21	41	03
23	00	22	40	12
24	00	23	39	20
25	00	24	38	28
26	00	25	37	37
27	00	26	36	45
28	00	27	35	53
29	00	28	35	02
30	00	29	34	10
31	01	00	33	18

Hours

Hours Min.	&c.	*Sun's*	*Long.*	Hours Min.	&c.	*Sun's*	*Long.*
h	d	'	"	h	d	'	"
'	'	"	'''	'	'	"	'''
1	00	02	28	31	01	16	23
2	00	04	56	32	01	18	51
3	00	07	24	33	01	21	19
4	00	09	51	34	01	23	47
5	00	12	19	35	01	26	15
6	00	14	47	36	01	28	43
7	00	17	15	37	01	31	11
8	00	19	43	38	01	33	39
9	00	22	11	39	01	36	06
10	00	24	38	40	01	38	44
11	00	27	06	41	01	41	02
12	00	29	34	42	01	43	30
13	00	32	02	43	01	45	58
14	00	34	30	44	01	48	25
15	00	36	58	45	01	50	53
16	00	39	25	46	01	53	21
17	00	41	53	47	01	55	49
18	00	44	21	48	01	58	17
19	00	46	49	49	02	00	44
20	00	49	17	50	02	03	12
21	00	51	45	51	02	05	40
22	00	54	13	52	02	08	08
23	00	55	40	53	02	10	36
24	00	59	08	54	02	13	03
25	01	01	36	55	02	15	31
26	01	04	04	56	02	17	59
27	01	06	32	57	02	20	27
28	01	09	00	58	02	22	55
29	01	11	27	59	02	25	23
30	01	13	55	60	02	27	51

The Moon's mean Motion.

	Moon's Long.				Moon's Node.			
Years of Chriſt	s	d	'	"	s	d	'	"
1701	10	15	20	00	04	27	23	20
1801	08	23	10	25	05	13	12	05
1901	07	01	00	50	07	29	00	50
2001	05	08	51	15	03	14	49	35
Years	Moon's Long.				Moon's Node.			
Com.	s	d	'	"	s	d	'	"
1	04	09	23	03	00	19	19	43
2	08	18	46	07	01	08	39	26
3	00	28	09	10	01	27	59	04
4	05	20	42	49	02	17	22	03
5	10	00	05	52	03	06	41	46
6	02	09	28	55	03	26	01	29
7	06	18	51	59	04	15	21	12
8	11	11	25	37	05	04	44	05
9	03	20	48	41	05	24	03	49
10	08	00	11	44	06	13	23	33
11	00	09	34	48	07	02	43	16
12	05	02	08	27	07	22	06	09
13	09	11	31	30	08	11	25	52
14	01	20	54	34	09	00	45	35
15	06	00	17	37	09	20	05	18
16	10	22	51	16	10	09	28	12
17	03	02	14	19	10	28	47	56
18	07	11	37	22	11	18	07	39
19	11	21	00	26	00	07	27	22
20	04	13	34	05	00	26	50	15
40	08	27	08	10	01	23	40	31
60	01	10	42	15	02	20	30	46
80	05	24	16	20	03	17	21	01
100	10	07	50	25	04	14	11	15

Months	Moon's Long.				Moon's Node.			
	s	d	'	"	s	d	'	"
January	00	13	10	35	00	00	03	11
February	02	01	38	41	00	01	41	41
March	02	10	35	01	00	03	10	38
April	03	29	03	07	00	04	49	08
May	05	04	20	37	00	06	24	27
June	06	22	48	43	00	08	02	56
July	07	28	06	14	00	09	38	16
Auguſt	09	16	34	19	00	11	16	47
Septemb.	11	05	02	25	00	12	55	16
October	00	10	19	55	00	14	19	55
Novemb.	01	28	48	01	00	16	09	01
Decemb.	03	04	05	32	00	17	44	25

Days	Moon's Long.				Moon's Node.			
	s	d	'	"	s	d	'	"
1	00	13	10	35	00	00	03	11
2	00	26	21	10	00	00	06	21
3	01	09	31	45	00	00	09	32
4	01	22	42	20	00	00	12	43
5	02	05	52	55	00	00	15	53
6	02	19	03	30	00	00	19	04
7	03	02	14	05	00	00	22	14
8	03	15	24	40	00	00	25	25
9	03	28	35	15	00	00	28	36
10	04	11	45	50	00	00	31	46
11	04	24	56	25	00	00	34	57
12	05	08	07	00	00	00	38	08
13	05	21	17	35	00	00	42	18
14	06	04	28	10	00	00	44	29
15	06	17	38	45	00	00	47	40
16	07	00	49	20	00	00	50	50
17	07	13	59	55	00	00	54	01
18	07	27	10	30	00	00	57	11
19	08	10	21	05	00	01	00	22
20	08	23	31	40	00	01	03	33

21

D.	s	d	'	"	s	d	'	"
21	09	06	42	15	00	01	06	43
22	09	19	52	50	00	01	09	54
23	10	03	03	25	00	01	13	05
24	10	16	14	00	00	01	16	15
25	10	29	24	35	00	01	19	26
26	11	12	35	10	00	01	22	37
27	11	25	45	45	00	01	25	47
28	00	08	56	20	00	01	28	58
29	00	22	06	55	00	01	32	09
30	01	05	17	31	00	01	35	19
31	01	18	28	06	00	01	38	30

Hours Min. &c.	*Moon's Long.*			*Moon's Node.*	
h	d	'	"	'	"
'	'	"	"'	"	"'
1	00	32	56	00	08
2	01	05	53	00	16
3	01	38	49	00	24
4	02	11	46	00	32
5	02	44	42	00	40
6	03	17	39	00	48
7	03	50	35	00	56
8	04	23	32	01	04
9	04	56	28	01	12
10	05	29	25	01	19
11	06	02	21	01	27
12	06	35	18	01	35
13	07	08	14	01	43
14	07	41	10	01	51
15	08	14	07	01	59
16	08	47	03	02	07
17	09	20	00	02	15
18	09	52	56	02	23
19	10	25	53	02	31

h	'	"	"	"	'''
20	10	58	49	02	39
21	11	31	46	02	47
22	12	04	42	02	55
23	12	37	39	03	03
24	13	10	35	03	11
25	13	43	32	03	19
26	14	16	28	03	27
27	14	49	25	03	34
28	15	22	21	03	42
29	15	55	17	03	50
30	16	28	14	03	58
31	17	01	10	04	06
32	17	34	07	04	14
33	18	07	03	04	22
34	18	39	59	04	30
35	19	12	55	04	38
36	19	45	52	04	46
37	20	18	48	04	54
38	20	51	45	05	02
39	21	24	41	05	10
40	21	57	38	05	18
41	22	30	54	05	26
42	23	03	31	05	34
43	23	36	27	05	42
44	24	09	24	05	50
45	24	42	20	05	58
46	25	15	17	06	06
47	25	48	13	06	14
48	26	21	10	06	22
49	26	54	06	06	30
50	27	27	03	06	38

h	′	″	″	″	‴
51	27	59	59	06	46
52	28	32	56	06	54
53	29	05	52	07	01
54	29	38	49	07	08
55	30	11	45	07	16
56	30	44	42	07	24
57	31	17	38	07	32
58	31	50	34	07	40
59	32	23	31	07	48
60	32	56	27	07	56

One Inſtance will be ſufficient to ſhew the Uſe of the Tables in finding the Place of the Sun and Moon, as 1736, *March* 15, at Midnight incluſive, being Leap Year.

The Sun's Longitude.

	s	d	′	″
1701	9	20	43	50
20	00	00	09	04
15	11	29	22	27
Mar.	01	29	08	20
15	00	14	47	05
	00	04	10	46
Equation			4	
Velocity			4	
	00	04	18	46

Moon's

	Moon's Long.				*Moon's Node.*			
	s	d	'	''	s	d	'	''
1701	10	15	20	00	04	27	23	20
20	04	13	34	05	00	26	50	15
15	06	00	17	37	09	20	05	18
March	02	10	35	01	00	03	10	38
15	06	14	54	03	00	00	47	40
	05	24	40	46	06	06	29	29
Lun.Line			31	30				
	05	25	12	16				
Equa.			10					
Velocity		9	50					
Equ.Ve.	06	05	12	16				
Subtract.			40					
	06	04	32	16				

the Node is found by continually ſubtracting.

9. Experience muſt prove the Truth of this Scheme, which is founded on continual Obſervation, and if there be any Improvements made, it muſt be done by the ſame, which I hope will not be rejected without ſufficient Trial, for if it is, I am confident the Moon's Place will never be found.

I ſhall conclude with one Obſervation, that this Scheme accounts for one Irregularity,

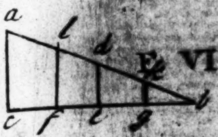
gularity, which appears in the Motion of the Sun or Earth; which is the Proceſſion of the Equinox; for if the Sun be imagin'd to move $50''$ above the Ecliptick, and never to cut it, as in (*Pl.* 1. *Fig.* 5.) where *eq* is the Equator, *ec* the Ecliptick and *lm*, the Line of the Sun, when the Sun comes to *m* it is $50''$ to the *Weſt* of the Ecliptick, which it loſes every Revolution; and thus it will recede through the Equator in 25920 Years, agreeable to the great *Platonic* Year, and the Stars will proceed to the *Weſt*; then the Sun by this, is always $50''$ higher than the Ecliptick, which maks its Declination in the Summer Solſtice to $100''$ or $1'\ 40''$ more than in the Winter: Hence likewiſe the Sun moves ſlower in the Summer than in the Winter by $200''$ or $3'\ 20'' = 7$ Minutes in Time, as is plain from the Nature of the Line, and what is ſaid about the Line of the Moon.

And thus, by repeated Obſervations, may ſome of the intricate Motions of the Heavens be diſcovered, which is but very little to what the Knowledge of Nature is. God to diſcover more the Depth of his own Perfections, to humble the aſpiring Pride and Vanity of Mankind, and give us greater Ideas of his infinite Wiſdom and Power, hath conceal'd theſe Things from the deepeſt Search of Mankind; all that they can do is to erect Schemes in their Imagination to ſolve ſome viſible Difficulties in the Motions of the World, without knowing any Thing of the internal Make or Eſſence of it; and if the viſib leNature is ſo deep and intricate, what muſt the inviſible God and Creator of all Nature be? With what Humility therefore ought we to think of him, and never mention his Holy Name, but with the greateſt Reverence! *Which, that we may all do, God of his infinite Mercy grant, through Jeſus Chriſt our Lord.* Amen.

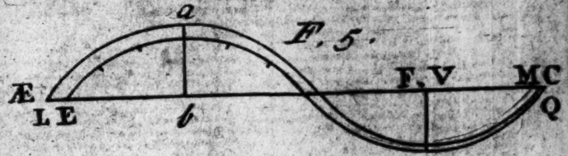
Laus Deo uni Trinoque.

FINIS.

The Circle Squared.



F. VI



F. 5.

F. IIII

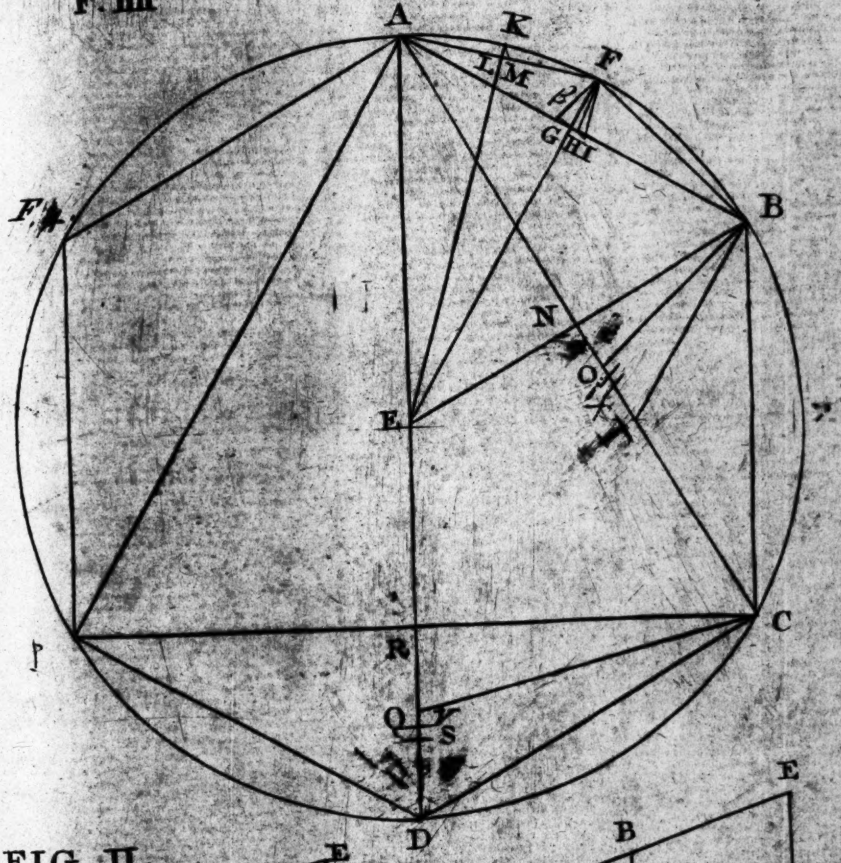
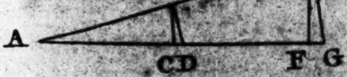


FIG. II.



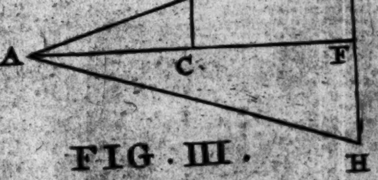
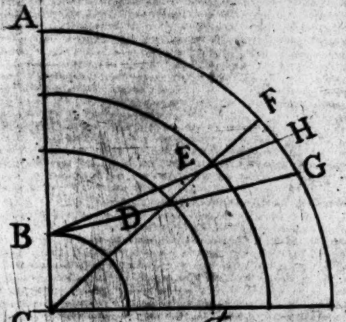
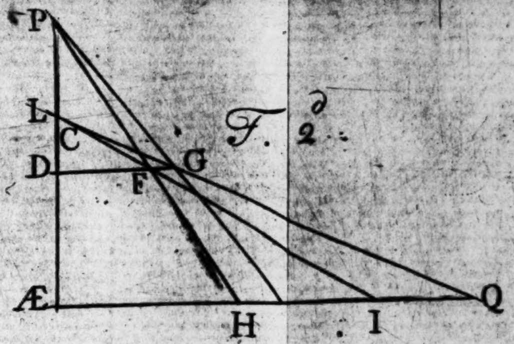


FIG. III.

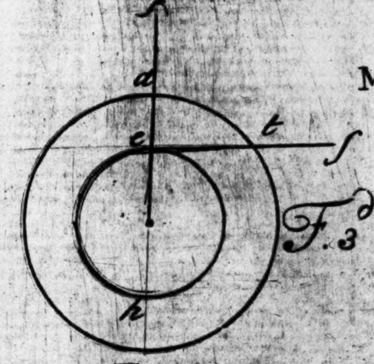
By *Richard Locke*.



F. 1st

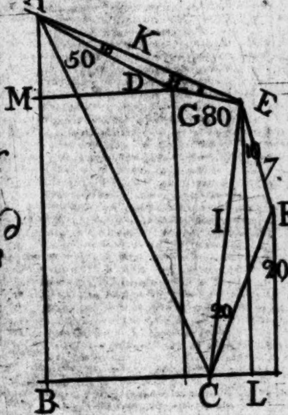


F. 2d

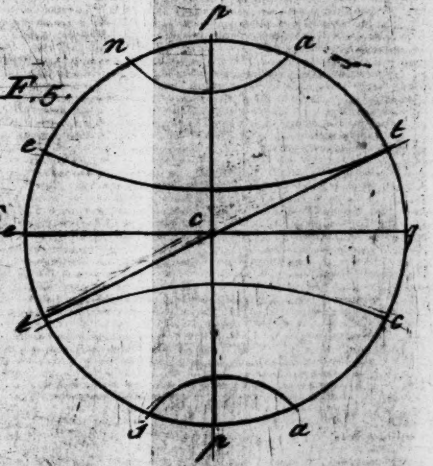


F. 3d

F. 4.



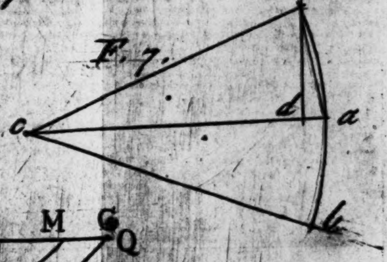
F. 5.



The Moon's Way in July 1732

Plate 2d

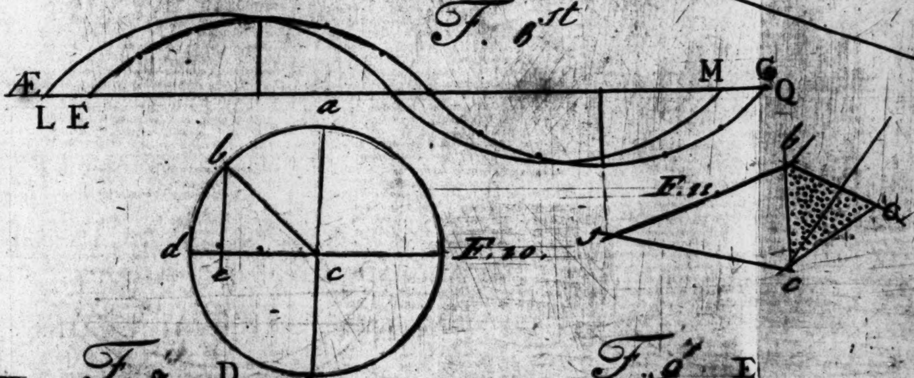
F. 7.



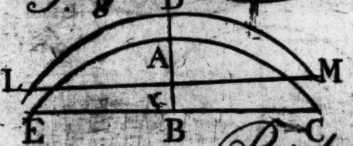
F. 6st

F. 10.

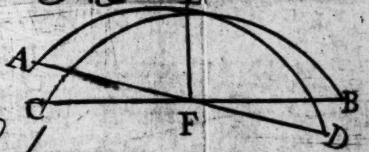
F. 11.



F. 8



F. 9



Richard Locke
AUTHOR

T. Hatton Sc

Fig. 1.

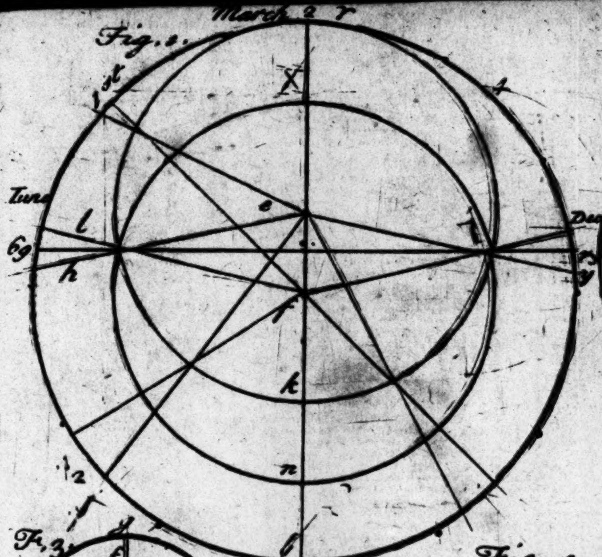
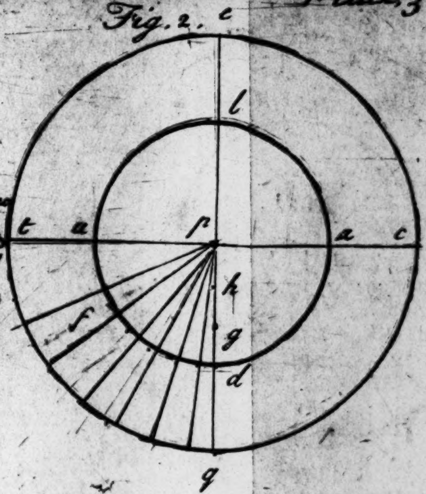


Fig. 2.



F. 3.

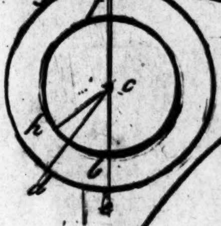
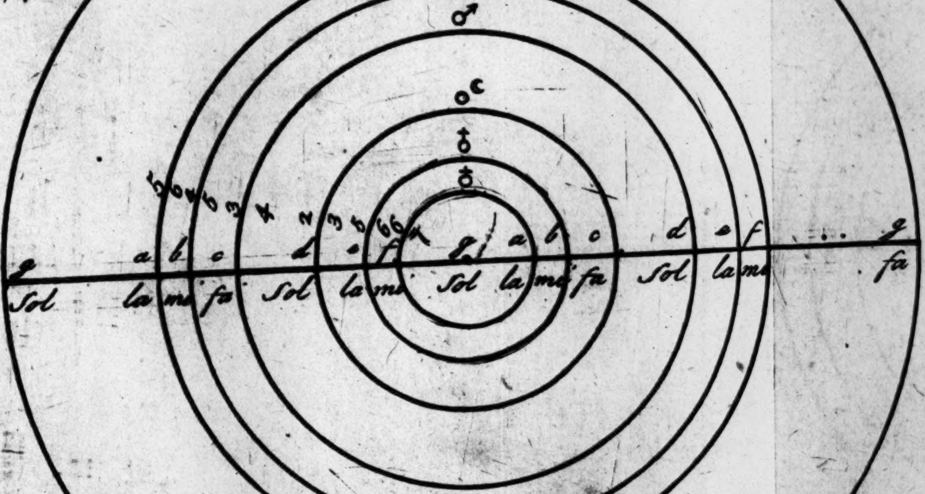
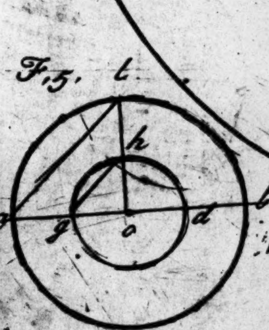


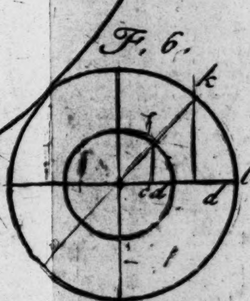
Fig. 4.



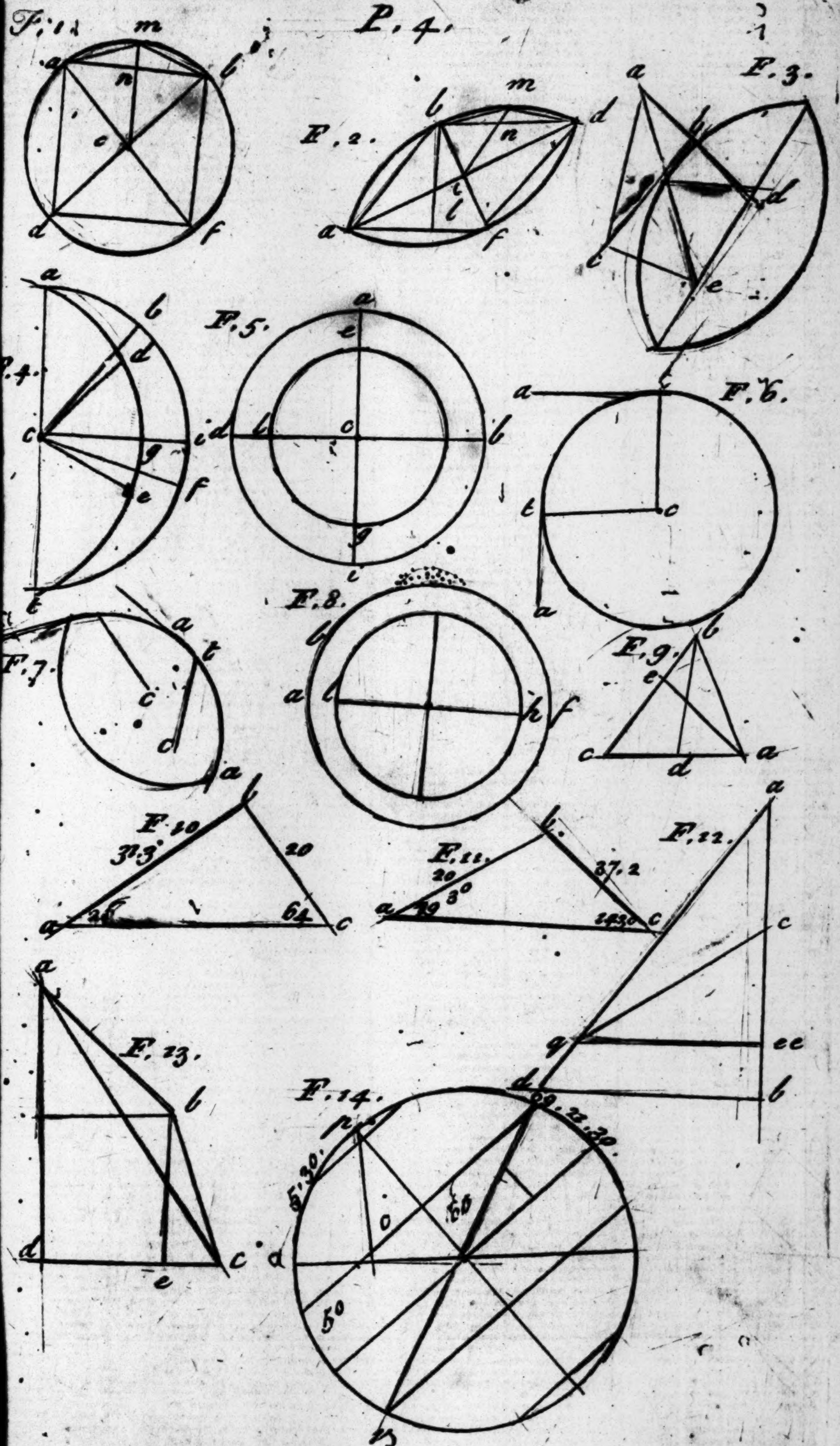
F. 5.



F. 6.



P. 4.


F. 1.
m
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F. 2.
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F. 4.
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F. 5.
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F. 6.
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